How Much Trading Volume is Too Much?

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ABSTRACT

Is there excessive trading volume in financial markets? Recent proposals to tax transactions, limit short selling, or restrict clienteles for particular financial products reflect such a view but beg the fundamental question of how much trading is optimal for an economy. We show that the optimal level of volume done purely for risk-sharing cannot be determined from the real economy alone, dictating that there is no natural bound on trading volume. We analyze how a variety of restrictions on trading influence welfare. Our results provide an economic basis for evaluating the desirability of policy proposals to limit trading activity.

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How much trading volume is too much?

The Harkin-DeFazio bill will place a speculation fee of 0.03 percent on credit default swaps, derivatives, stocks, bonds, and other financial transactions…. The Harkin-DeFazio bill would reduce gambling on Wall Street, encourage the financial sector to invest in the productive economy, and significantly reduce the deficit without harming average Americans.

---Statement of Senator Bernie Sanders on Harkin-Defazio financial transactions tax at http://www.sanders.senate.gov/imo/media/doc/0228-TransTax.pdf

The idea that finance is too large, that it should be restricted in some ways, is becoming more commonplace. Arguments that the financial system is simply outsized relative to the real sector, that banks are behemoths that should be restricted in scale, that trading volume is far too excessive have been put forth by a host of commentators ranging from the European Central Bank to Bernie Sanders.[1] Even financial economists have entered the fray, with recent research by Rajan [2011], Cochrane [2013], Greenwood and Scharfstein [2013], and Zingales [2015] addressing aspects of this debate. While interesting in their own right, these debates take on added consequence when they translate into specific proposals to restrict financial activity. Transactions taxes, short sale constraints and CDS clientele restrictions are but three examples of such size-linked restrictions.

In this paper, we develop an analytically tractable approach for investigating the role of trading volume and the interactions between the financial sector and the real sector in affecting welfare. Our model, which builds from the literature first developed by Radner [1972] and Hart [1975], provides an innovative way to address issues by using a representative agent in an effectively complete market structure. We then use this framework to consider the topic at hand: how much trading volume is too much? Our focus is not on whether trading volume could be excessive (in our view, it surely can). Rather, we address the more concrete question of can policy makers recognize the optimal level of productive volume and so avoid placing restrictions that do more harm than good? Such a bottom-up approach was a mainstay of economic analysis

[1] See, for example, Speech by Lorenzo Bini Smaghi, "Has the Financial Sector Grown Too Big?" April 15, 2010, available at http://www,ecb.europa.eu/press/key/date/2010/html/sp100415.en.html; or "Bernie Sanders on Financial Regulation" available at www.berniesanders.com.

in the past, but seems to have been overlooked in current deliberations on how best to control or curtail the financial markets.

Our analysis is based on a simple model in which trading occurs only for risk-sharing. Of course, trading volume arises for a variety of other reasons including speculation and disagreement, but the point we make here is that even in the absence of such motivations, trading volumes can be large relative to the size of the real economy.[2] We establish our results in a model of plans, prices, and price expectations (PPE) based on Radner [1972] in which traders have common expectations. We show that there is a representative agent for our economy and we use this technique to circumvent the non-existence of equilibrium problem that has limited prior work on this topic.

Why does any of this matter? At an academic level, understanding volume is fundamental for much of what we study in finance. As Hong and Stein [2003] observe "we find it hard to imagine a fully satisfying asset-pricing model – in either the rational or behavioral genre – that does not give a front-and-center role to volume." Our model shows that, even abstracting from speculative or disagreement motives for trade, trading volume related to risk-sharing motivations is not an artifact of the exchange process, but rather depends on both the risks in the real economy and on the structure and efficiency of available financial contracts. Hence, low volume is not necessarily "better" and high volume is not necessarily "worse" – the optimal amount of volume in terms of enhancing welfare is not calibrated on a linear metric. At a practical level, our results provide a necessary economic backdrop for evaluating the desirability of policy proposals designed to limit trading activity. Our results particularly highlight the challenge of separating "good" volume from "bad" volume, suggesting that how much volume is too much is not easily discerned.

1. Model

The economy we analyze is based on the model of plans, prices and price expectations in Radner [1972]. This standard model of trading in a sequence of markets is the basis for much of the research on sequential markets in economics and finance. We consider a pure exchange

[2] For an analysis of the effects of speculative motives on volume see Simsek [2013]. A classic paper on disagreement and volume is Hong and Stein [2007]. There is also an empirical literature looking at the effects of transactions taxes on volume and volatility, see for example Jones and Seguin [1997].

economy which lasts for two periods, $t = 1, 2$. There is uncertainty about which state of the world, $s = 1, \dots, S$, will be realized at date two. We let $p(s)$ denote the probability of state s and let $p = (p(1), \dots, p(S))$. There are L physical goods at date two in each state. We denote the prices of physical goods in state s by $q(s) \in \Delta^L = \{q \in R_+^L \mid \sum_l q_l(s) = 1\}$.

There are I consumers indexed by $i = 1, \dots, I$. Consumer i has an endowment of goods at date two in state s given by $e^i(s) \in R_{++}^L$; consumer i's endowment vector across states is denoted $e^i = (e^i(1), \dots, e^i(S))$. Consumers trade assets at date one in order to share the risk created by their random date two endowments. We assume that consumers have Bernoulli utility functions $u^i : R^L \to R$ which are strictly concave, strictly monotone, and continuous functions of date two consumption. These consumers have common, correct beliefs $p(s)$ about the probability of state s. So consumer i's expected utility of the date two consumption plan $x^i = (x^i(1), \dots, x^i(S)) \in R^{LS}$ is $\sum_s p(s) u^i(x^i(s))$.

At date one there are K securities markets open.[3] Each security pays off in one, and only one, good at date two and we let $j(k)$ be the good that security k pays in at date two.[4] For example, good one could be gold (which could be used as a numeriare as long as its equilibrium date two price is positive in every state) and there could be securities paying off in various amounts of gold in various states. Another security could pay a constant amount of some physical commodity across states; and yet another security could pay one unit of some commodity in one-and-only-one state. We let $A = [a_k(s)]$ represent the $K \times S$ matrix of security payoffs where $a_k(s) \geq 0$ is the amount of its good that security k pays per unit at date two if state s occurs. Consumers trade these securities to move wealth across states and thus they care about the $K \times S$ matrix of expected monetary returns $M = [a_k(s) q_{j(k)}(s)]$.

We assume that all consumers have correct and thus identical expectations of date two goods prices. This rational expectations hypothesis rules out speculative trade; in our model, consumers trade only to share risk. They do this by trading securities at time one and they use the

[3] To keep the analysis simple, and focused on asset trade, the only trade at date one is in assets.
[4] The assumption that each security is identified with a single good is made only for notational convenience.

monetary returns on these securities at time two to hedge the income risk created by their random endowments. We let $\pi = (\pi(1), \ldots, \pi(K))$ be the security price vector at date one. Security prices are normalized so that $\pi \in \Delta^K = \{\pi \in R_+^K \mid \sum \pi(s) = 1\}$.

Let z_k^i be i's purchase of security k and $z^i = (z_1^i, \ldots, z_K^i)$ be i's portfolio of securities. Consumer i's decision problem is to select a feasible consumption plan and security purchase to maximize expected utility subject to a sequence of budget constraints. Formally, this decision problem is

$$\max \sum_s p(s) u^i(x^i(s))$$

$$\text{s.t.}\ (x^i, z^i) \in$$

$$\{(x^i, z^i) \in R^{LS} \times R^K : \pi z^i = 0, \text{ and } q(s)x^i(s) \le q(s)e^i + z^i M(s), \text{ for all } s\},$$

where M(s) is the s'th column of M.

We let $D^i(\pi, q)$ be the set of optimal consumption plans and portfolios for consumer i. This set may be empty as to this point we have not made sufficient assumptions to insure that there are no arbitrage opportunities. Of course, arbitrage opportunities cannot exist in equilibrium; so if there are equilibrium prices there are no arbitrage opportunities at those prices. We next define what we mean by equilibrium.

Equilibrium

An equilibrium in this economy is a list of consumption plans and portfolios for the consumers, and prices for date two goods and (date one) securities such that date one security markets clear and date two goods markets would clear in each state at those goods prices. Note that in an equilibrium, consumers perfectly forecast date two goods prices and so they have common and correct expectations about the prices which would occur tomorrow. This is why in equilibrium trade in securities occurs only to share risk.

Formally, an **equilibrium of plans, prices and price expectations (PPPE)** (see Radner [1972]) is (π^*, q^*) and $(z^i, x^i)_{i=1}^I$ such that:

$$1.\ (z^i, x^i) \in D^i(\pi^*, q^*) \text{ for all i}$$

$$2.\ \sum_i z^i = 0$$

$$3.\ \sum x^i(s) = \sum e^i(s) \text{ for all s}.$$

PPPE need not exist or be optimal if they do exist. Radner [1972] sidesteps the existence problem by exogenously imposing a bound on securities trade. Hart [1975] shows, however, that Radner's bound is not innocuous. He constructs an economy in which a PPPE equilibrium does not exist without the bound. [5] To study the equilibrium volume of trade, we need an economy in which equilibrium exists without such a bound on trade. We do this by constructing an economy that has a representative consumer. To understand how our approach works, it is useful to first examine how Hart's non-existence example works (using our structure). First, note that our consumers trade securities only in order to move income across states to hedge their endowment risk. So what matters about security payoffs is the monetary returns matrix, M. If this matrix has rank S then the securities are equivalent to Arrow securities (Arrow [1964]) and securities can be used to move wealth across states to achieve any desired wealth profile. If M has rank less than S, then there are constraints (which can be derived from M) on the wealth profiles that can be achieved through security trade.

In Hart's example, if M has full rank, then markets are effectively complete and consumers move wealth across states so that date two goods prices are complete markets prices q^*; and if it has a lower rank (markets are incomplete) consumers cannot move wealth as freely and date two goods prices are not q^*. Non-existence of equilibrium occurs in this example because if prices at date two are expected to be q^* then M does not have full rank. Thus date two prices cannot be q^*. Alternatively, if consumers expect the prices that would occur with incomplete markets, then prices are such that M has full rank and equilibrium date two prices would have to be q^*. So in Hart's example there is no price expectation which is self-fulfilling and thus there is no equilibrium.

The relation between Hart's analysis and the bound on security trade imposed by Radner is that for a sequence of prices converging to q^*, markets are effectively complete but security trade grows arbitrarily large as consumers are using nearly linearly dependent securities to move wealth across states. Radner's bound on trade is thus binding in Hart's example and it restores equilibrium because of this.

[5] He also provides examples in which there are multiple Pareto-ranking equilibria or in which adding a security can make every trader worse off.

For our analysis, it's important that equilibrium exist (without having a bound on trade) and we insure this by constructing an economy with a representative consumer. Of course, the economy we discuss has multiple consumers, so that there is trade at date one, but for the purposes of determining date two prices the economy aggregates. Thus, the distribution of wealth across consumers at date two does not affect date two prices. We can determine those prices without reference to the structure of financial markets as those markets serve only to change the distribution of wealth across consumers at date two. There is no issue with existence of a PPPE in this economy because whether markets are complete (or not) is determined only by the rank of the monetary returns matrix evaluated at these **fixed** date two prices.

Although we primarily focus on complete markets, we make use of the fact that a large volume of trade is necessary to move wealth across states for securities that are nearly linearly dependent. In our analysis this occurs not because we are changing expectations about future prices (they are fixed at the equilibrium date two prices), but because we ask whether the size of trade can be determined by the real economy without reference to the financial market structure (among the class of effectively complete financial markets). As we demonstrate, the answer to this question is no---the optimal amount of trade in financial assets cannot be determined from knowledge of the real economy alone.

Complete Markets

We first note that for the real economy what matters about the financial market structure is the rank of the monetary returns matrix M. As long as M has full rank, financial markets are effectively complete in the sense that trade in financial markets at time one can be used to generate any desired wealth profile at time two. For any such financial market structure, equilibrium in the real economy (consumptions, prices of physical goods and consumer welfare) is independent of the market structure.

Our economy has complete markets if $K = LS$ with one security for each good in each state. In this case, consumers can directly trade all state contingent goods and there is an equilibrium in which no trade occurs at date two. Arrow [1963-64] observed that it is sufficient to have only $K = S$ (Arrow) securities with all securities paying off in a single good (that has positive price in each state at date two). Arrow's securities insure that the monetary returns matrix has rank S. To see why Arrow's argument works, consider the wealth profile (across states) that consumers would have in a complete markets equilibrium. Consumers can trade

Arrow securities to achieve this wealth profile (because M has full rank) and so the prices that will occur at date two are the complete markets prices. This argument also implies that any monetary returns matrix which has rank S at complete markets prices is equivalent to the monetary returns matrix resulting from Arrow securities and so it results in a complete markets equilibrium.

Remark 1: The following (effectively complete) security structures are all equivalent in the sense that they all lead to the same (set of) equilibria in the real economy (consumption plans for date two and prices of date two physical goods):

1. $K = LS$ with one security for each good in each state.
2. $K = S$ with all securities paying off in a single good (that has a positive price in each state in any complete markets equilibrium) and $A = I$, the $S \times S$ identity matrix.
3. Any security payoff matrix such that at all complete markets equilibrium prices M has full rank S.

If the economy has an effectively complete markets security structure it is equivalent to a complete markets economy in the sense that equilibrium consumptions and goods prices are complete markets prices. Then equilibrium security trades and prices can be derived to support those complete market equilibrium consumptions and goods prices. So standard results about the existence of a complete markets competitive equilibrium and the optimality of any such equilibrium immediately imply existence of an equilibrium of PPPE and its optimality. Our assumptions about utility functions and endowments are sufficient to guarantee the existence of a complete markets equilibrium. This immediately implies:

Remark 2: If securities are such that at complete markets prices M has rank S, then there is an equilibrium of PPPE and any equilibrium consumption allocation is Pareto Optimal.

2. Trade in a Simple Economy

In this section, we examine a leading example of the economy introduced in section 1. We use this simplified economy to make various points about the equilibrium and optimal volume of trade in financial markets.

This economy must have multiple consumers as otherwise there cannot be any trade in the financial markets at time one. It also has a representative consumer so that the trade in financial markets does not affect date two prices. The existence of a representative consumer requires strong assumptions on the preferences of the actual consumers, but it is a standard approach to asset pricing in both macroeconomics and finance. Our leading example has two goods, $L = 2$, two states with equal probability, $S = 2$, and two consumers, $I = 2$. These consumers have endowments that are state dependent:

$$e^1(1) = (1,0), e^1(2) = (0,1), e^2(1) = (0,1), e^2(2) = (1,0).$$

Note that in this economy there is no aggregate risk as the total endowment is $(1,1)$ in each state. However, each consumer faces individual endowment risk as well as wealth risk if the goods have differing prices.

Our consumers have a common utility function. In each state s this function is

$$u^i(x^i(s)) = \alpha \ln x_1^i(s) + \beta \ln x_2^i(s)$$

with $\alpha > 0$, $\beta > 0$, $\alpha + \beta = 1$ and $\alpha > \beta$.[6]

Representative Consumer and Date Two Prices

With these utility functions and endowments, date two prices are easy to determine. Suppose consumer i has income $W^i(s)$ in state s, then consumer i's demands for goods 1 and 2 in state s are:

$$x_1^i(s) = \alpha W^i(s) / p_1(s) \quad \text{and} \quad x_2^i(s) = \beta W^i(s) / p_2(s)$$

Market clearing at date two in state s requires

$$\alpha(W^1(s) + W^2(s)) = p_1(s) \quad \text{and} \quad \beta(W^1(s) + W^2(s)) = p_2(s).$$

Note also that $W^1(s) + W^2(s) = 1$ for all s and so $p_1(s) = \alpha$ and $p_2(s) = \beta$ in each state.[7]

[6] The assumption that α and β are not equal is used to induce wealth risk. Although we use these specific Cobb-Douglas utility functions, all we need for our analysis is indirect utility functions of the Gorman form with a common coefficient on wealth, so that aggregate demand can be expressed as a function of aggregate wealth. This is still a restrictive assumption, so we stay with the simple Cobb-Douglas utility functions in the text.

[7] Our structure results in constant date two prices because of the representative consumer and constant aggregate endowments across states. All we need for our analysis is equilibrium date two prices that are independent of the

Note that these prices do not depend on the structure of asset markets. In any equilibrium, and for any date one assets, these are the date two prices that will occur. So we can examine the welfare consequences of financial asset market structures without needing to be concerned about their effect on date two prices. There are welfare consequences of differing financial market structures because consumers use these markets to share risk.

Equivalent Asset Markets

We begin by considering the simplest effectively complete asset markets. Suppose that there are two securities with security one paying one unit of good one in state one and nothing in state two, and security two paying one unit of good two in state two and nothing in state one. At equilibrium goods prices the monetary returns matrix is

$$\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

This matrix has full rank and so the markets are effectively complete and equilibrium consumptions are given by their complete markets values. It is easy to see that in equilibrium, consumers trade securities so that they each have an income of $1/2$ in each state and equilibrium security prices are $\pi^1 = \alpha$ and $\pi^2 = \beta$. The equilibrium volume and dollar volume of security trade is:

1. Volume: $(\alpha - \beta)/2\alpha$ for security 1 and $(\alpha - \beta)/2\beta$ for security 2
2. Dollar volume: $(\alpha - \beta)/2$ for security 1 and $(\alpha - \beta)/2$ for security 2
3. Total dollar volume of security trade: $(\alpha - \beta)$.

This is the natural dollar volume of security trade *with assets perfectly tailored to sharing the risk faced by the consumers*. To see this, note that without security trade consumer one would have an income of α in state one and an income of β in state two. With effectively complete markets, consumer one wants, and can achieve, an income of $1/2$ in each state. So he must

distribution of wealth and thus independent of the structure of security markets. We use the simple structure in the text to make our examples easy to follow.

move $\alpha - 1/2$ dollars of income out of state one and $1/2 - \beta$ dollars of income into state two. That is, the value of his total trade must be $(\alpha - 1/2) + (1/2 - \beta) = \alpha - \beta$; the total dollar volume of equilibrium security trade given above.

No Natural Bound on the Volume of Security Trade

It may be tempting to conclude from the observation above that $\alpha - \beta$ is the "natural dollar volume" of security trade in this economy; or, at least, that there is some natural bound on security trade. This is false.

Remark 3: There is no bound that can be placed on the equilibrium volume or dollar volume of trade independently of the structure of (effectively complete) security markets. As the equilibrium is Pareto Optimal this also implies that there is no independent-of-security-structure bound that can be placed on the volume of trade needed to support the Pareto Optimal allocation of consumption goods that arises with complete markets.

The proof of this claim is simple. Suppose that there are two securities with security one paying 1 unit of good 1 in state 1 and $0 \le \delta < 1$ units of good 1 in state 2, and security two paying 1 unit of good 2 in state 2 and $0 \le \delta < 1$ units of good 2 in state 1. Note that if $\delta = 1$ then security trade does not move income across states and markets are not effectively complete.

These assets induce a monetary returns matrix which evaluated at equilibrium date two prices is

$$\begin{bmatrix} \alpha & \delta\alpha \\ \delta\beta & \beta \end{bmatrix}$$

For all $\delta \ne 1$ this matrix has full rank and so the markets are effectively complete and we get the complete markets equilibrium consumptions. Calculation shows that equilibrium security trade is described by:

1. Volume: $(\alpha - \beta)/2\alpha(1-\delta)$ for security 1 and $(\alpha - \beta)/2\beta(1-\delta)$ for security 2.
2. Dollar volume: $(\alpha - \beta)/2(1-\delta)$ for security 1 and $(\alpha - \beta)/2(1-\delta)$ for security 2.

3. Total dollar volume of security trade: $(\alpha-\beta)/(1-\delta)$.

Note that as $\delta \to 1$ equilibrium security trade measured in units of securities or in dollars diverges. Thus, knowledge of the real economy (endowments, utility functions and the probability on states) is not sufficient to bound the optimal volume of security trade. This occurs even though security trade in this economy occurs only for risk sharing. In this example, security trade is large for δ near one because the securities are not very effective in moving wealth across states, and as a result, consumers need to trade large amounts of them in order to share risk.

There is no Natural Bound on Short Sales

It is natural to suspect that someone shorting a security is doing so in order to speculate on the good in which the security pays.[8] In our economy securities are short- lived, so no one could be speculating on changes in the price of any security, but if we interpret short selling as holding a negative amount of the security, then short selling certainly occurs in our economy even though no one is trading for speculative purposes. It's obvious that a ban on short selling is harmful in this economy. Any risk sharing requires someone to go short as our securities are in zero net supply. So a ban on short selling would lead to no security trade and an equilibrium in which consumers are forced to bear idiosyncratic risk. The resulting no-short-selling-equilibrium is clearly Pareto inferior to the unconstrained, effectively complete markets equilibrium.

There is also no benign bound on short selling that can be determined from the real economy. This point is immediate from the equilibrium calculations above. With the security structure in Remark 3, consumer one is shorting security one and going long in security two and the amounts of these trades are the negative of the equilibrium volume of trade in security one and the equilibrium volume of trade in security two. The dollar volumes are also the negative of the value of the dollar volumes of trade in securities one and two, respectively. So as δ goes to 1 the amount by which consumer one goes short in security one diverges. Similarly, the amount by which consumer two goes short in security two diverges. Any bound on short selling will be binding for some $\delta < 1$ and it will result in consumers unnecessarily bearing idiosyncratic risk.

No Natural Clientele

[8] There is a large literature looking at the impact of short sale constraints when traders have differences of opinion or differential information. See, for example, Jarrow [1980], Allen and Gale [1994], or Hong and Stein [2003].

Another common misunderstanding about security markets is that securities have ``natural clienteles'' and that only the natural clientele for a security should be trading that security, or at least that anyone else trading the security is speculating rather than trading to reduce risk. In our economy, the natural clientele for a security could be interpreted as those consumers who are endowed with risky amounts of the good in which the security pays. These consumers hold risk which they might naturally want to reduce by trading in securities which payoff in the risky goods they begin with. However, because consumers trade securities to move income across states, full risk-sharing may not be possible if each consumer's trading is limited to securities that payoff in goods he owns.

Remark 4: Define the Natural Clientele for a security to be consumers who are endowed with the good in which the security pays off. Then restricting trade in a security to its Natural Clientele can result in a Pareto inferior allocation of consumption goods.

The proof of this claim is also simple. Suppose that person one is endowed only with good 1 and person two is endowed only with good 2 as follows:

1. $e^1(1) = (1,0), e^1(2) = ((1-\alpha)/\alpha, 0)$.
2. $e^2(1) = (0,1), e^2(2) = ((2\alpha - 1)/\alpha, 1)$.

Note that aggregate endowments are constant at one unit of each good in each state.
Suppose, as in our first example, that there are two securities with security one paying one unit of good one in state one and nothing in state two, and security two paying one unit of good two in state two and nothing in state one. At equilibrium date two prices the monetary returns matrix is again

$$\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

This matrix has full rank and so the markets are effectively complete and equilibrium consumptions are given by their complete markets values. In equilibrium, consumers trade securities so that they each have an income of $1/2$ in each state and equilibrium security prices are $\pi^1 = \alpha$ and $\pi^2 = \beta$. Consumer one's equilibrium security trades are:

1. $z_1^1 = (1-2\alpha)/2\alpha$ of security one,

2. $z_2^1 = (2\alpha - 1)/\beta$ of security two.

In equilibrium, consumer one trades security 2, which pays off only in units of good 2, even though he has no endowment of good 2 in any state. The consumer does this to move income across states---it is the distribution of income across states that matters and not the amounts of securities or the goods they are tied to that is important.

A restriction saying that consumer one cannot trade security two means that he also cannot trade security one as otherwise his period one budget constraint would be violated. So if security trade is restricted to Natural Clienteles there is no security trade, we have autarky, and there is no risk sharing. In the equilibrium that occurs with trading restricted to Natural Clienteles, the consumers are forced to bear idiosyncratic risk which is Pareto inferior to the complete markets (perfect risk sharing) equilibrium consumptions that would otherwise occur.

Innovation in Securities Can Reduce or Increase the Volume of Security Trade

That innovation can lead to an increase in the volume of security trade and an increase in welfare is obvious.[9] The simplest example is to compare the economy without security markets to one with effectively complete security markets. Without security markets there is no security trade and in equilibrium consumers bear idiosyncratic risk. With effectively complete security markets, there is trade (and its size cannot be bounded without knowledge of exactly what securities are available) and the resulting consumption allocation is Pareto superior to the one without security trade. So:

Remark 5: Adding new securities can increase the volume of security trade and increase welfare

Innovation in securities can also reduce the amount of trade. To see this we modify the security structure in the proof of Remark 3 so that security one pays one unit of good one in state one and nothing otherwise while security two continues to payoff in both states. Markets are still effectively complete and calculation shows that equilibrium security trade is now described by:

1. Volume: $(\alpha - \beta)(1+\delta)/2\alpha$ for security 1 and $(\alpha - \beta)/2\beta$ for security 2.

[9] Our results here are consistent with Allen and Gale [1994] who developed the idea that financial innovation develops largely to facilitate risk sharing. Other researchers have examined various motivations for financial innovation and its implications for welfare and market outcomes. For an excellent survey, see Duffie and Rahi [1995].

2. Total dollar volume of security trade: $(\alpha-\beta)(1+\delta)/(\alpha+\beta(1+\delta))$.

The total dollar volume of security trade is less than $(\alpha-\beta)/(1-\delta)$ for any $\delta<1$. So this innovation has reduced security trade relative to its value using the security structure in Remark 3. It's useful to note that this reduction in trade occurs because security one is now more effective at transferring risk between states one and two. Previously it paid-off in both states one and two, so using it to move wealth into state one also moved wealth into state two and this had to be compensated for by shorting security two. Revised security one's effectiveness in moving wealth across states is a direct result of it being riskier after the innovation as it now pays-off only in state one. Simple calculations show that the variance of the returns on security one increases from $((1-\delta)/2)^2$ to $((1+\beta\delta)/2)^2$ after the innovation. So we have:

Remark 6: Modifying a security to make it riskier can reduce the dollar volume of security trade.

The security innovation used in the argument for Remark 6 has no welfare consequences as security markets are effectively complete both before and after the innovation, and so equilibrium consumptions are unaffected by the innovation. Innovations that make securities riskier can have positive welfare consequences. To see this, suppose that initially security one pays one unit of good one in both states and security two pays one unit of good two in both states. Then as goods prices are independent of states, these securities do not allow any transfer of wealth across states; the monetary returns matrix does not have full rank. These securities are, of course, risk free as each of them has a monetary return that is state-independent, but it's exactly this independence that makes them useless for risk sharing. If they are modified using the $\delta\neq1$ perturbation as in the proof of Remark 3 then they are riskier, but they are now effective at transferring wealth across states. Now markets are complete and the resulting equilibrium allocation is a Pareto improvement on the one that occurs with risk-free securities.

3. Policy debates and political debates

Financial markets, and in particular securities trading, play a role in facilitating risk-sharing across individuals. When done effectively, such trading improves welfare. Our analysis demonstrates several salient facts about such trading: there is no natural bound on the trading volume needed to effectuate optimal risk-sharing; short selling may be a necessary ingredient to do so; and there is no natural trading clientele. We also demonstrated that adding securities could increase volume and increase welfare, and that making securities riskier can reduce volume and increase welfare. Overall, our model suggests that trading volume plays a complex role in the financial markets, and that discerning "excessive" trading volume from knowledge of the real economy alone is not actually feasible.

These findings may be particularly useful when applied to policy questions. Restrictions placed on trading volumes, arising, for example, from short sale restrictions, clientele limitations, or transactions taxes are often positioned as effective policy instruments to remove the wrong kind of volume from the market. Transactions taxes, for example, have been suggested as way to "reduce gambling on Wall Street, encourage the financial sector to invest in the productive economy, and significantly reduce the deficit without harming average Americans."[10] Germany's imposition of clientele limitations on credit default swaps on European sovereign debt was presumably intended to remove excessive speculation.[11] Short sale restrictions imposed during the financial crisis reflected similar goals. The problem, of course, is that such restrictions are blunt instruments, affecting all volume – both the "good" kind and the "bad" kind. Our analysis here shows that discerning the optimal level of even (presumably good) risk-sharing related volume is not feasible, making it inevitable that some valuable trading will be restricted.

Does this mean that such restrictive proposals should not be undertaken? Not necessarily. We think that addressing such questions is beyond the purview of policy research and instead lies more in the realm of political debates. Policy research can delineate the economic implications of proposed actions, and predict (and, in the case of empirical research, estimate) the potential costs and benefits of specific actions. These findings, in turn, should inform the political debate. Our analysis here, for example, suggests limitations on the efficacy and feasibility of these volume-linked policies, but the political debate may also involve issues

[10] See statement of Bernie Sanders at http://www.sanders.senate.gov/imo/media/doc/0228-TransTax.pdf See also John Carney, "What would happen if Bernie Sanders Taxed Wall Street?, Wall Street Journal, Feb. 13, 2016.
[11] See Floyd Norris, "Naked Truth on Default Swaps", New York Times, May 20, 2010.

such as whether the benefits derived from using transactions taxes to finance college educations is sufficient to justify the potential welfare losses arising from the tax's effects on trading. Without the underlying policy research delineating these losses, however, political debates cannot hope to escape being polemic discourses long on potential benefits and short on actual costs.

Finally, we note that it's natural to be concerned about the generality of our results given the very simple model we use. However, our claims about the lack of a natural limit on the volume of trade, the need for short-selling and the lack of natural clienteles for securities are true regardless of the number of consumers, states or goods in the economy. We chose to present the results in a simple model so as to not obscure our points about trade. Our analysis does, however, depend on being able to determine future goods prices independently of the security structure. This occurs if aggregate demand at date two can be written as a function of aggregate wealth; that is if the distribution of wealth across consumers does not affect equilibrium date two prices. We do not think that wealth effects could change our qualitative conclusions, but they could certainly make it more difficult to demonstrate them cleanly.

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