Differential Access to Price Information in Financial Markets

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Abstract

Recently exchanges have been directly selling market data. We analyze how this practice affects price discovery, the cost of capital, return volatility, market liquidity and welfare. We show that selling price data increases the cost of capital and volatility, worsens market efficiency and liquidity, and discourages the production of fundamental information relative to a world in which all traders freely observe prices. Generally allowing exchanges to sell price information benefits exchanges and harms liquidity traders. Overall, our results show that allowing exchanges to sell market data, rather than requiring it to be made freely available to the public, is undesirable.

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Price information plays a crucial role in securities markets. Because of this fundamental role, securities regulators in the U.S. mandated the formation of a consolidated tape to provide real time information on every trade execution in the U.S. markets. Trade and quote data also provide a major source of revenue for stock exchanges.[1] The U. S. exchanges jointly own the Consolidated Tape Association and the Consolidated Quote Association which together sell the comprehensive quote and trade data available on the tape. Exchanges share in the tape revenue depending upon the volume of trades and the production of quotes in each market. Recently, however, some exchanges have found a way to supplement their tape revenue by directly selling market data.[2] Purchasers of the data benefit in that they see the data before it appears on the consolidated tape, while exchanges benefit by essentially selling the same data twice, albeit at different speeds. The effects of this practice on the market are, however, not so clear.

In this paper we investigate what happens when some traders purchase access to market information before other traders. There are many instances where this occurs in markets.[3] First, purchasing fast data (along with practices such as co-location of trading terminals with exchange computers) gives rise to a practice called "latency arbitrage" whereby some

[1]Stock exchange revenues come primarily from trading fees, listing fees and market data. For the New York Stock Exchange/Euronext, high market volumes resulted in market data revenues exceeding listing revenues in 2008 and 2009. In 2011, market data products contributed 371 million dollars to total revenue.

[2]The NYSE sells a variety of direct data feeds via its products NYSE Best Quote, NYSE Open Book, and NYSE Amex Best quote. NASDAQ sells its trade and quote data directly under the product name NASDAQ ITCH. Arcapelago (a part of NYSE/Euronext) also sells trade and quote. BATS and Direct Edge provide data feeds from their markets but currently do not charge for this data. In Europe, the London Stock Exchange, the Deutshe Borse and virtually all of the major exchanges sell price and trade data. The Deutshe Borse, for example, recently introduced a new data product "MIFID Post Trade" providing trade prices and volumes for securities traded on the Frankfurt stock exchange and the Tradegate Exchange. The Hong Kong Exchange has experimented with provision of free real-time prices and is now moving to a system in which information vendors will be charged for data access.

[3]We develop our analysis in terms of equity markets, but the general principles we illustrate should apply equally well to other asset markets such as futures markets. The keys to our analysis are that trade takes place in a rational expectations equilibrium, that the market may sell access to prices and that traders have the ability to develop at a cost information about the future value of the assets they are trading.

traders are able to see market data and trade before other traders.[4] Second, in particularly volatile periods delays in data transmission sometimes arise (see the Summary Report of the Joint CFTC-SEC Advisory Committee on Emerging Regulatory Issues written in response to the "Flash Crash" of May 6, 2010) and not all traders experience the same delay.[5] Third, price data on some trades (odd-lot trades are a particularly important example) is produced by the market, but it is not provided to the public through the consolidated tape.[6]

Our goal is to shed light on the effects of differential access to price information on traders' behavior and on the performance of the market. We show that differential access generally increases the cost of capital and volatility, and reduces market efficiency and liquidity relative to an economy in which all traders can observe price data. Selling differential access to price information obviously benefits the exchange, but it harms liquidity traders. Its effect on rational traders is complex and depending on the details of the economy they can be better-off or worse-off if the exchange is allowed to sell price information. This occurs because traders who acquire price information benefit from being able to take advantage of liquidity traders but they have to pay the exchange for this privilege.

The negative effects on price discovery, on the cost of capital and on volatility arise because "price-informed" traders benefit at the expense of "price-uninformed" traders who in turn scale back their holdings to offset this increased risk.[7] The negative effect on market liq-

[4]See Hendershott and Moulton (2008) and Easley, Hendershott, and Ramadorai (2010) for analyses of latency issues in equity markets. Hasbrouck and Saar (2011) provide a discussion and empirical investigation of low-latency trading. They point out that the quest for relative speed and the trading advantage it produces is not new and that the issue is not the magnitude of clock time advantage (now in milliseconds), but whether it produces a strategic advantage in trading. High frequency issues are also addressed in Kandel and Tkatch (2008).

[5]The SEC report (page 76) noted also that at the height of the flash crash NYSE quotes in 1665 securities had average delays to the CQS of over 10 seconds. Between 2:45 and 2:50 pm, over 40 of these stocks had average delays greater than 20 seconds. See also Peterson (2010).

[6]See O'Hara, Yao and Ye (2011) for analysis.

[7]Huang and Wang (2009) in their model of market crashes show that, in the presence of participation costs, liquidity shocks lead to price declines because of their asymmetric affects on risk. They focus on

uidity arises because “price-informed” traders make prices more responsive to fundamentals and thus less responsive to noise trading. Thus, relative to a world in which all traders observe prices, allowing exchanges to sell price information serves mainly to enhance exchange profit at the expense of increasing the equity risk premium and reducing market liquidity.

Selling price data also has important effects on the information structure of financial assets. We show that if traders have to pay for both price data and fundamental data then for reasonable parameter specifications no one purchases fundamental data without also purchasing price data. That is, there are no traders who chose to become informed of only fundamental data. As a result, relative to an economy in which all traders can freely observe price data, selling price data curtails the production of fundamental information, thereby harming price informativeness. This, in turn, increases both the cost of capital and return volatility, and it typically lowers liquidity.

A number of authors, for example Admati and Pfleiderer (1986, 1988, 1990), Fishman and Hagerty (1995), Allen (1999), Garcia and Vanden (2009), and Garcia and Sangiorgi (2011), have considered the issue of selling information in financial markets, although typically in the context of analysts selling fundamental information to other traders. In our setting, the exchange does not trade on the data it creates, so the strategic decision to trade for one’s self or to sell data to others that this literature focuses on does not arise. Our analysis extends this literature, however, by showing how price data can substitute for fundamental data, thereby introducing another dimension into the trade-offs facing holders (and sellers) of fundamental data. Our analysis further complements this literature by exploring the implications of selling price data for market outcomes (i.e. market efficiency, the cost of capital,

episodic problems due to liquidity shocks. We focus on ongoing problems due to differences in access to information, but the asymmetric affects on risk are similar and in our analysis they lead to a higher cost of capital.

return volatility, liquidity, and trader welfare), an issue not addressed by earlier literature. Finally, in our analysis the precision of the price data being sold by the exchange is endogenous and is determined in equilibrium by the amount of private fundamental information which, in turn, is affected by the equilibrium precision of the price data. By contrast, in the earlier literature the precision of the data to-be-sold is exogenous (in both cases the value of the information is endogenous).

There is also a large literature examining differential access to information in the context of insider trading (see, for example, Glosten (1989), Fishman and Hagerty (1990), and Leland (1992)). In our analysis, all traders have equal access to information regarding underlying asset values, but some traders have "inside information" about market data. In common with the earlier literature about insider trading, we find that this form of differential information creates important welfare, liquidity, price discovery and cost of capital effects. In particular, our finding that greater availability of price data lowers the cost of capital complements Leland's finding that more insider trading can have a similar positive effect in the market.

A recent paper by Cespa and Foucault (CF) (2008) also investigates the role of price data in securities markets. Our analysis and theirs differ in important modeling choices, in the primary questions asked, and in the results and policy implications. CF find that the Pareto optimal market structure is either fully opaque or has limited transparency. They show how this optimal level of transparency can be achieved by charging a fee (a Pigovian tax) for price information designed to curb excessive acquisition of price information. Our focus is not on Pareto optimality, but rather we characterize the equilibrium in the market for information and the resulting equilibrium in the market for securities when an exchange can sell information. Our primary conclusion is that markets outcomes are "best" in the sense of having the lowest cost of capital and return volatility as well as the greatest liquidity and informational efficiency if regulators require exchanges to provide price data freely to all traders (i.e. markets are fully transparent).

In deriving the equilibrium when exchanges are allowed to sell price information, we solve the intermediate problem of characterizing the effects of differential information about prices on the cost of capital and market liquidity. Here our results are the opposite of those in CF, and this divergence from two main differences in modeling choices. First, CF use a random endowments model in which they assume that traders transact in advance of knowing their endowments. Trading then allows traders to hedge this endowment risk, and it is this hedging demand that makes greater opacity desirable. In our economy, liquidity traders introduce the necessary randomness. Trade occurs because of differences in information and the randomness in prices induced by the trades of liquidity traders. A second difference in the models arises from assumptions regarding per capita supply. In CF, this supply is zero and consequently their analysis cannot address issues related to the cost of capital (which is always zero in their model under any market structure). By contrast, the cost of capital plays a pivotal role in our analysis because changes in the availability of price data change the risk confronting traders not holding this data, and their subsequent demand to be compensated changes equilibrium prices and liquidity.

We believe our framework is a more natural setting to investigate issues related to the desirability of allowing exchanges to sell data, an issue that is now a major policy debate both in the U.S. and in Europe. In Europe, there is no consolidated tape, so exchange data products give traders information not available elsewhere. In the US, exchange data products give traders information before it appears on the tape. For some data, the delay may be measured in seconds (or parts thereof), but other data (such as odd lot prices or information on hidden orders) are never reported to the tape. Given the speed at which high frequency trading now occurs, any delay now matters for the market equilibrium.[8] Our model demonstrates why this matters.

We develop the implications of our research for this debate and our recommendations for regulatory policy in more detail later in the paper. We argue that the "fairness" criterion the Securities and Exchange Commission (SEC) has proposed to evaluate the issue of differential access is misguided. We propose an alternative criterion based on market properties to guide regulatory decision-making in this area. Using this criterion, we argue that some types of data selling should be prohibited.

The paper is organized as follows. In the next section we set out the model. Sections II and III respectively describe the equilibrium in the asset and price information markets and derive implications for positive market outcomes. The policy implications of various requirements about price disclosures are discussed in Section IV. In Section V we generalize the analysis to make the acquisition of fundamental information endogenous and describe the resulting complex effects on the equilibrium and welfare. Section VI discusses the implications of our analysis for the regulatory debate about access to price information in financial markets. Finally, the Appendix contains generalizations and extensions of the model.

I. The Model

There are two tradable assets in our economy: one risk-free asset, cash, which has a constant value of 1; and one risky asset which has a price of $\tilde{p}$ per unit and an uncertain future value

[8] Direct evidence on the importance of speed is starkly illustrated by the construction of the Hibernian Express, the first new transatlantic cable in 10 years. When it is completed in 2013, the new $300 million cable will reduce the speed of transmitting orders between London and New York to 59.6 milliseconds from the current level of 64 milliseconds. The project is privately funded and its customers are large hedge funds engaged in high frequency trading.

denoted $\tilde{v}$. We assume that $\tilde{v} \sim N(\bar{v}, 1/\rho_v)$ with $\bar{v} > 0$ and $\rho_v > 0$.[9]

There are two types of traders: rational traders and liquidity traders. There is a continuum $[0, 1]$ of rational traders with CARA utility functions with common coefficient of risk aversion γ. Each rational trader is endowed with cash only and for simplicity, we suppose their endowment is zero. Liquidity traders provide the randomness that is necessary to make our rational expectations equilibrium partially revealing in the sense that they supply $\tilde{x}$ units of the risky asset per capita to the market. We do not endogenize the behavior of liquidity traders, rather we view them as individuals who are trading to invest new cash flows or to liquidate assets to meet unexpected consumption needs.[10]

We assume that $\tilde{x} \sim N(\bar{x}, 1/\rho_x)$ with $\bar{x} > 0$ and $\rho_x > 0$. The assumption that the mean per capita supply of the asset is positive is important for our results. If instead it is 0, then on average there is no aggregate risk to be borne, and in equilibrium no one will be rewarded for bearing it. We believe that the pricing of aggregate risk is important, so we focus on the case in which it exists. One can view this risky asset as a proxy for the stock market, and in this case aggregate uncertainty is unavoidable.

Initially we assume that each trader is endowed with a private signal:

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N(0, 1/\rho_\varepsilon) \text{ and } \rho_\varepsilon > 0. \tag{1}$$

We will later (in Section V) endogenize the decision to acquire private signals.

Rational traders are further categorized into two groups according to whether they pay

[9]Generalizing our analysis to many independent securities is straightforward as under our structure asset demands are independent across assets. Including correlated assets is also possible, but here we focus on the single asset case for clarity of exposition.

[10]The noise induced by liquidity traders can equivalently be viewed as random (from the point of view of rational traders) float of securities. Alternatively, it is possible to model the decision problem of traders who experience endowment shocks which would endogenize our liquidity traders. We do not do this as it greatly complicates the analysis without providing additional insights.

the profit-maximizing exchange at a price of $q > 0$ to observe the current stock price $\tilde{p}$. If a trader acquires $\tilde{p}$, then she can submit orders conditional on $\tilde{p}$ and $\tilde{s}_i$, that is, she submits a demand schedule $DI\left(\tilde{p}, \tilde{s}_i\right)$. If a trader does not acquire $\tilde{p}$, then she can only submit orders conditional on $\tilde{s}_i$—$DU\left(\tilde{s}_i\right)$—essentially a market order. We call those traders purchasing $\tilde{p}$ *price-informed*; the traders not purchasing $\tilde{p}$ are called *price-uninformed*. We suppose that there is a fraction $\mu > 0$ of price-informed traders. In Section III we endogenize the decision to acquire the contemporaneous price.

It is worth pointing out that in our model the exchange is selling access to contemporaneous price-information. Traders who purchase this information can condition their demand for the risky asset on the equilibrium price; while those who do not purchase access to price-information cannot condition on the contemporaneous price. This is of course an abstraction of how modern stock markets function. In reality, no one can condition on the equilibrium price (and the fact that it is an equilibrium) and actually there is no single equilibrium price for a stock. We use this abstraction to provide some insight into the effects of selling differential access to price information in the cleanest possible setting. In Appendix E we consider a dynamic model in which old price data is freely available and the exchange sells access to the most recent past price information. That analysis is substantially more complex, but the results are the same as those in the text.

II. Financial Market Equilibrium

We begin by deriving a rational expectations equilibrium (REE) in the stock market with a fixed fraction μ of price-informed traders. In an equilibrium, per capita demand for the risky asset must equal per capita supply. So the market clearing condition is:

$$\int_0^{\mu} DI\left(\tilde{p};\tilde{s}_i\right)di+\int_0^{1-\mu} DU\left(\tilde{s}_i\right)di=\tilde{x}. \tag{2}$$

We show in Proposition 1 that there is a price function such that if all traders conjecture that prices are determined by this function then market clearing prices are in fact determined by this function. The functional form we derive is:

$$\tilde{p}=\alpha+\beta\tilde{v}-\lambda\tilde{x}. \tag{3}$$

We define the cost of capital, return volatility and market liquidity as usual as:

$$CC\equiv E\left(\tilde{v}-\tilde{p}\right),\ RetVol\equiv\sigma\left(\tilde{v}-\tilde{p}\right)\ \text{and}\ liquidity\equiv\lambda^{-1}.$$

That is, the cost of capital CC is the expected difference between the cash flow generated by the risky asset and its price. This difference arises from the compensation required to induce rational traders to hold the risky asset. The return on the risky asset is $(\tilde{v}-\tilde{p})$, and thus its volatility can be measured by $\sigma\left(\tilde{v}-\tilde{p}\right)$. Market liquidity measures the market depth: a smaller λ means that liquidity trading has a smaller price impact, and so the market is deeper. Much of our positive analysis is focused on the effect of the sale of price information on these three statistics and on price discovery (i.e., market efficiency, which will be introduced shortly below).

A. The Equilibrium Price Function

By equation (3), observing the price is equivalent to observing the following signal about the asset payoff $\tilde{v}$:

$$\tilde{s}_p = \frac{\tilde{p} - \alpha + \lambda \bar{x}}{\beta} = \tilde{v} - m^{-1} \left(\tilde{x} - \bar{x} \right), \tag{4}$$

where

$$m = (\beta / \lambda) \tag{5}$$

denotes the "market efficiency" or "price informativeness" measure, as is standard in noisy rational expectation equilibrium models (e.g., Kyle (1989), Brunnermeier (2005), Peress (2010), and Ozsoylev and Walden (2011)).

The demand functions and indirect utility functions for the traders in our economy take on standard forms that are derived in Appendix A. For price-informed traders and price-uninformed traders these functions are, respectively:

$$DI\left(\tilde{p}; \tilde{s}_i\right) = \frac{E\left(\tilde{v} | \tilde{p}, \tilde{s}_i\right) - \tilde{p}}{\gamma Var\left(\tilde{v} | \tilde{p}, \tilde{s}_i\right)}, \tag{6}$$

$$VI\left(\tilde{p}, \tilde{s}_i\right) = -\exp\left\{ -\gamma \bar{W}_i + \gamma q - \frac{\left[E\left(\tilde{v} - \tilde{p} | \tilde{p}, \tilde{s}_i\right)\right]^2}{2Var\left(\tilde{v} | \tilde{p}, \tilde{s}_i\right)} \right\}, \tag{7}$$

$$DU\left(\tilde{s}_i\right) = \frac{E\left(\tilde{v} - \tilde{p} | \tilde{s}_i\right)}{\gamma Var\left(\tilde{v} - \tilde{p} | \tilde{s}_i\right)}, \tag{8}$$

$$VU\left(\tilde{s}_i\right) = -\exp\left\{ -\gamma \bar{W}_i - \frac{\left[E\left(\tilde{v} - \tilde{p} | \tilde{s}_i\right)\right]^2}{2Var\left(\tilde{v} - \tilde{p} | \tilde{s}_i\right)} \right\}, \tag{9}$$

where $\bar{W}_i$ is the initial wealth of trader i.

To write these demand and indirect utility functions in explicit form, it is useful to compute the conditional moments (from various trader's points of view) of the distribution of the asset's value. It follows immediately from the conjectured form of the price function and Bayes' rule that these moments are:

$$\begin{aligned} E\left(\tilde{v}|\tilde{p},\tilde{s}_i\right) &= \frac{\rho_v \bar{v} + \rho_\varepsilon \tilde{s}_i + m^2 \rho_x \tilde{s}_p}{\rho_v + \rho_\varepsilon + m^2 \rho_x}, \qquad (10) \\ Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right) &= \frac{1}{\rho_v + \rho_\varepsilon + m^2 \rho_x}, \qquad (11) \end{aligned}$$

$$\begin{aligned} E\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right) &= -\alpha + (1-\beta)\frac{\rho_v \bar{v} + \rho_\varepsilon \tilde{s}_i}{\rho_v + \rho_\varepsilon} + \lambda \bar{x}, \qquad (12) \\ Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right) &= (1-\beta)^2 \frac{1}{\rho_v + \rho_\varepsilon} + \lambda^2 \frac{1}{\rho_x}. \qquad (13) \end{aligned}$$

Inserting these moments into the trader's demand functions, we have:

$$\begin{aligned} DI\left(\tilde{p},\tilde{s}_i\right) &= \frac{\rho_v \bar{v} + \rho_\varepsilon \tilde{s}_i + m^2 \rho_x \tilde{s}_p - \left(\rho_v + \rho_\varepsilon + m^2 \rho_x\right)\tilde{p}}{\gamma}, \\ DU\left(\tilde{s}_i\right) &= \left(\frac{1}{\gamma}\right) \frac{-\alpha + (1-\beta)\frac{\rho_v \bar{v} + \rho_\varepsilon \tilde{s}_i}{\rho_v + \rho_\varepsilon} + \lambda \bar{x}}{(1-\beta)^2 \frac{1}{\rho_v+\rho_\varepsilon} + \lambda^2 \frac{1}{\rho_x}}. \end{aligned}$$

Substituting these demands into the market clearing condition and comparing coefficients, we have the following system defining the two unknown coefficients β and λ in the conjectured price function:

$$\begin{aligned} \beta &= \frac{\mu \rho_\varepsilon + (1-\mu)(1-\beta)\frac{\rho_\varepsilon}{\rho_v+\rho_\varepsilon}\left[(1-\beta)^2 \frac{1}{\rho_v+\rho_\varepsilon} + \lambda^2 \frac{1}{\rho_x}\right]^{-1}}{\mu\left[\left(\rho_v + \rho_\varepsilon + m^2\rho_x\right) - m^2 \rho_x \beta^{-1}\right]}, \qquad (14) \\ \lambda &= \frac{\gamma}{\mu\left[\left(\rho_v + \rho_\varepsilon + m^2\rho_x\right) - m^2 \rho_x \beta^{-1}\right]}. \qquad (15) \end{aligned}$$

Using the above system we can solve for β and λ. The idea is to first express β and λ as functions of m and to get one equation in terms of m. This equation is linear and can be solved for m analytically. The following proposition shows that there is an equilibrium price function of the conjectured form and it provides a characterization of this equilibrium price function.

Proposition 1 *Suppose $\mu > 0$. There exists a partially revealing rational expectations equilibrium, with price function*

$$\tilde{p} = \alpha + \beta\tilde{v} - \lambda\tilde{x},$$

where

$$\beta = \frac{\gamma\mu^{-1}m + m^2\rho_x}{\rho_v + \rho_\varepsilon + m^2\rho_x}, \tag{16}$$

$$\lambda = \frac{\gamma\mu^{-1} + m\rho_x}{\rho_v + \rho_\varepsilon + m^2\rho_x}, \tag{17}$$

$$\alpha = (1-\beta)\,\bar{v} + \lambda\bar{x} - \gamma\bar{x}\left[\mu Var^{-1}\left(\tilde{v}|\tilde{p},\tilde{s}_i\right) + (1-\mu)\,Var^{-1}\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)\right]^{-1},$$

with

$$m = \frac{\mu\rho_\varepsilon\left(\gamma^2 + \mu\rho_v\rho_x + \mu\rho_\varepsilon\rho_x\right)}{\gamma\left(\gamma^2 + \mu^2\rho_v\rho_x + \mu\rho_\varepsilon\rho_x\right)}, \tag{18}$$

$$\frac{1}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)} = \frac{\mu^2\rho_x\left(\rho_v+\rho_\varepsilon\right)}{\gamma^2 + \mu^2\rho_x\left(\rho_v+\rho_\varepsilon\right)}\frac{1}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)}, \tag{19}$$

$$\frac{1}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)} = \rho_v + \rho_\varepsilon + m^2\rho_x. \tag{20}$$

B. The Impact of the Fraction of Price-Informed Traders

We next use the expression for the equilibrium price to analyze how the fraction of price-informed traders affects price informativeness, the cost of capital, return volatility and liquidity.

Price Informativeness We begin by analyzing the effect of μ on price informativeness as this is the driving force for our reslts on the cost of capital, return volatility, and liquidity. Taking the derivative with respect to μ in equation (18) in Proposition 1, we have

$$\frac{\partial m}{\partial \mu} = \left(\frac{\rho_\varepsilon}{\gamma}\right) \frac{\left(\gamma^2 + \mu \rho_\varepsilon \rho_x\right)^2 + \gamma^2 \mu \left(2 - \mu\right) \rho_v \rho_x + \mu^2 \rho_v \rho_\varepsilon \rho_x^2}{\left(\gamma^2 + \mu^2 \rho_v \rho_x + \mu \rho_\varepsilon \rho_x\right)^2} > 0. \tag{21}$$

Thus, increasing the fraction of price-informed traders improves price informativeness. This is because price-informed traders trade more aggressively on their fundamental information than price-uninformed traders and thus the presence of more price-informed traders will bring more information into the equilibrium price.

Corollary 1 *As more traders become price-informed, the price becomes more informative; that is* $\frac{\partial m}{\partial \mu} > 0$.

The Cost of Capital By Proposition 1, the cost of capital is

$$CC \equiv E\left(\tilde{v} - \tilde{p}\right) = \frac{\gamma \bar{x}}{\mu Var^{-1}\left(\tilde{v}|\tilde{p}, \tilde{s}_i\right) + \left(1 - \mu\right) Var^{-1}\left(\tilde{v} - \tilde{p}|\tilde{s}_i\right)}. \tag{22}$$

So, the cost of capital increases in traders' risk aversion γ, the asset supply $\bar{x}$, and the average risk that traders are exposed to per unit of the asset $\left[\mu Var^{-1}\left(\tilde{v}|\tilde{p}, \tilde{s}_i\right) + \left(1 - \mu\right) Var^{-1}\left(\tilde{v} - \tilde{p}|\tilde{s}_i\right)\right]^{-1}$.

Increasing the fraction of price-informed traders reduces the cost of capital through two effects, one direct and one indirect. The direct effect occurs because price-informed traders demand more of the risky asset than price-uninformed traders, as their knowledge of the price reduces the riskiness of the asset for them. So, as more traders become price-informed, the equilibrium price of the risky asset increases. The indirect effect occurs through the positive impact of a more informative price on the demands of all traders. For price-informed traders, a more informative price helps them to better forecast the future asset value, reducing their risk of trading the risky asset. For price-uninformed traders, a more informative price implies that the prevailing price is closer to the fundamental of the asset, and hence they too face less risk. Thus, a more informative price causes all traders to demand more of the risky asset thereby increasing in its price. Formally, we have the following corollary.

Corollary 2 *As more traders become price-informed, the cost of capital decreases; that is* $\frac{\partial CC}{\partial \mu} < 0$.

Return Volatility As we increase the fraction of price-informed traders, the price will reveal more information about $\tilde{v}$, causing the difference between the future value of the asset and its current price to be smaller for any amount of noise in the economy. Thus the return on the risky asset, $(\tilde{v} - \tilde{p})$, is smaller which reduces return volatility. Formally, we have the following corollary.

Corollary 3 *As more traders become price-informed, return volatility decreases; that is* $\frac{\partial RetVol}{\partial \mu} < 0$.

Liquidity By equation (17) in Proposition 1,

$$liquidity \equiv \frac{1}{\lambda} = \frac{\overbrace{\rho_v + \rho_\varepsilon + m^2\rho_x}^{\text{Uncertainty Reduction Effect}}}{\underbrace{\gamma\mu^{-1}}_{\text{Size Effect}} + \underbrace{m\rho_x}_{\text{Adverse Selection Effect}}}.$$

The fraction of price-informed traders, μ, affects each of the three components of liquidity. The first effect is a direct effect arising from the term $(\gamma\mu^{-1})$ in the denominator. We call this the "size effect" because it describes the direct impact of the size of the fraction of price-informed trader population on liquidity. An increase in μ will positively affect liquidity through this effect. When μ is small, the price will be very responsive to liquidity trading $\tilde{x}$, that is the market will be illiquid, as changes in liquidity trader demand will have to be absorbed by the few price-informed traders and this can only occur through large price changes.

The other effects on liquidity are indirect effects that occur through price informativeness m. Changes in μ affect liquidity differently through these two effects. The "uncertainty reduction effect" captures the fact that that increasing μ will increase price informativeness, causing price-informed traders to trade more aggressively, making price more responsive to fundamentals $\tilde{v}$ than to liquidity trading $\tilde{x}$, and thus improving liquidity. In contrast, the "adverse selection effect" captures the fact that the improved price informativeness will make it possible for price-informed traders to draw stronger inferences from price, making their demands more responsive to price changes induced by liquidity trading, and thus making the market less liquid.

Although the impact of μ on liquidity is complex we can show that if traders are sufficiently risk averse (γ large) or if there is sufficient risk in the economy (ρ_v or ρ_ε is small), then as the fraction of price-informed traders increases, market liquidity increases.

Corollary 4 *If γ is sufficiently large, or if ρ_v or ρ_ε are sufficiently small, then if more traders are price-informed, the market becomes more liquid. That is, $\frac{\partial liquidity}{\partial \mu} > 0$ for large γ and small ρ_v or ρ_ε.*

Figure 1 illustrates Corollaries 1-4 for the calibrated parameter values given by Table I. Note that for Corollary 4 to hold, we only need $\gamma = 2$ in Figure 1. Thus the sufficiently large value of γ in this Corollary does not have to be all that large. The other parameters in Table

I are borrowed from Leland's (1992) calibration. Since Leland's original calibration is based on annual S&P500 data, our results should also be interpreted on an annual basis.[11] The expected payoff of the risky asset $\bar{v}$ is normalized to 1. The ex ante payoff precision ρ_v is 25, which gives an annual volatility of 20%. We follow Gennotte and Leland (1990) in setting the rational trader's signal-to-noise ratio as 0.2, i.e., $\frac{\rho_\varepsilon}{\rho_v} = 0.2$, implying that the precision of the private signal is $\rho_\varepsilon = 5$. We normalize the per capita supply of the risky asset to 1, so that $\bar{x} = 1$. The precision of the liquidity trading is set to 4 at each date, that is, $\rho = 4$, which corresponds to an annual volatility of liquidity trading equal to 50% of total supply.

[INSERT FIGURE 1 AND TABLE I HERE]

III. Buying and Selling Price Information

The equilibrium fraction of traders who purchase price information is determined by comparing the indirect utility of a price-informed trader with that of a price-uninformed trader. The benefit, $B(\mu)$, of purchasing price data is the difference of these two indirect utilities. Calculation shows that this benefit is:

$$B(\mu) = \frac{1}{2\gamma} \log \left[\frac{Var(\tilde{v} - \tilde{p}|\tilde{s}_i)}{Var(\tilde{v} - \tilde{p}|\tilde{p}, \tilde{s}_i)} \right]. \tag{23}$$

Observing the market clearing price has two potential benefits: (i) the trader knows the prevailing price at which his trade will execute and hence the immediate price risk is reduced; and, (ii) the price contains information about the future fundamental value $\tilde{v}$, and hence it is

[11] As we explained in the introduction, the delay of price data in reality (and hence the period length in our model) depends on specific scenarios, varying from milliseconds to infinity. We here choose an annual frequency for simplicity.

useful for forecasting. We can decompose the total benefit into these two benefits as follows:

$$\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}=\underbrace{\frac{Var\left(\tilde{v}|\tilde{s}_i\right)}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)}}_{\text{Value Forecast}}\times\underbrace{\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}{Var\left(\tilde{v}|\tilde{s}_i\right)}}_{\text{Price Risk}}.$$

Using the characterization of the market clearing price function in Proposition 1 we know that these two benefits are

$$\text{Value Forecast Effect}=1+\frac{m^2\rho_x}{\rho_v+\rho_\varepsilon},$$

$$\text{Price Risk Effect}=\frac{\gamma^2+\mu^2\rho_x\left(\rho_v+\rho_\varepsilon\right)}{\mu^2\rho_x\left(\rho_v+\rho_\varepsilon+m^2\rho_x\right)}.$$

The "value forecast effect" is increasing in μ: As more traders observe the price, the price becomes more efficient, and thus more valuable. This leads to a learning complementarity generated by the feedback loop of observing price information. Formally, $\left(1+\frac{m^2\rho_x}{\rho_v+\rho_\varepsilon}\right)$ is increasing in μ. This effect does not occur in analyses of the sale of exogenous information where the precision of the information (rather than its value in the marketplace) is independent of the fraction of traders who purchase it.[12]

The second effect, the "price risk effect", is a substitution effect; that is, $\frac{\gamma^2+\mu^2\rho_x(\rho_v+\rho_\varepsilon)}{\mu^2\rho_x(\rho_v+\rho_\varepsilon+m^2\rho_x)}$ is decreasing in μ. This occurs because an increase in μ will cause the equilibrium price to be closer to the fundamental value thus decreasing the benefit of reducing price risk (recall that price informativeness increases with μ).

Overall, the second effect dominates, so that observing price information is a substitute,

[12]For example, this effect is absent in Admati and Pfleiderer (1986) where the information sold has exogenous precision. To see this, let $\tilde{y}=\tilde{v}+\tilde{\eta}$ be the exogenous signal sold by an information seller, where $\tilde{\eta}\sim N\left(0,1/\rho_\eta\right)$ and $\tilde{\eta}$ is independent of $\tilde{v}$. Then the value forecast effect is $\frac{Var(\tilde{v}|\tilde{s}_i)}{Var(\tilde{v}|\tilde{y},\tilde{s}_i)}=1+\frac{\rho_\eta}{\rho_v+\rho_\varepsilon}$, which is independent of the fraction μ of traders who purchase data $\tilde{y}$.

that is, $B(\mu)$ decreases with μ. Formally, using the expressions of the value forecast effect and the price risk effect, we can compute $B(\mu)$ as follows:

$$B(\mu) = \frac{1}{2\gamma} \log \left[1 + \frac{\gamma^2}{\mu^2 \rho_x (\rho_v + \rho_\varepsilon)} \right] \tag{24}$$

which is decreasing in μ.

For any fraction μ of price-informed traders the maximum amount that the exchange can charge these traders to observe the price is the benefit they receive, $B(\mu)$. The revenue the exchange receives if fraction μ are price-informed is thus $B(\mu)\mu$. The exchange's decision problem is:

$$\max_{\mu \in (0,1]} B(\mu)\mu = \frac{1}{2\gamma} \max_{\mu} \left\{ \mu \log \left[1 + \frac{\gamma^2}{\mu^2 \rho_x (\rho_v + \rho_\varepsilon)} \right] \right\}.$$

There are two possibilities for the optimal (from the point of view of the exchange) μ^*: $\mu^* = 1$ or $0 < \mu^* < 1$. The following proposition provides a characterization of the parameters for which it is optimal for the exchange to sell price information to everyone, i.e. μ^* is 1.

Proposition 2 *Let z^* be the constant solving the equation $\log(1+z^*) = \frac{2z^*}{1+z^*}$. If $\frac{\gamma^2}{\rho_x(\rho_v+\rho_\varepsilon)} \geq z^*$, the equilibrium fraction of price-informed traders is 1 (i.e., $\mu^* = 1$) and the equilibrium price of information is $q^* = B(1)$. Otherwise, the optimal solution is $\mu^* = \frac{\gamma}{\sqrt{z^* \rho_x(\rho_v+\rho_\varepsilon)}}$ and the equilibrium price of information is $q^* = \frac{1}{2\gamma}\log(1+z^*) = \frac{z^*}{\gamma(1+z^*)}$.*

Proposition 2 shows that when the risk aversion parameter γ is sufficiently large, the exchange will set $\mu^* = 1$. This occurs as in this case price information is so valuable to traders that the exchange can sell it to everyone at a high price. Under the technology parameter configuration in Table I, the lower bound for γ characterized by Proposition 2 is equal to 21.69. Figure 2 provides an illustration of this result.

[INSERT FIGURE 2 HERE]

IV. The Impact of Selling Price Information

In this section we evaluate the effects of allowing the exchange to sell access to the price rather than requiring that it be provided to everyone for free. To do this we consider two alternative economies. In Economy F all traders can observe the price for *free* perhaps because of SEC regulations requiring equal access to market prices. Economy D is the economy analyzed in the previous sections in which traders are *differentially* informed and the exchange chooses the profit maximizing price to charge for access to market prices. The welfare consequences of requiring the exchange to provide free access are not obvious because market prices will differ in the two economies. At this point in the analysis we keep access to private signals about the value of the risky asset exogenous. Of course, selling access to market prices affects the value of private information about the risky asset, and in the next section we endogenize the decision to acquire private information.

A. Benchmark Economy: Economy F

We use Economy F in which all traders can observe the equilibrium price for "free" as the benchmark economy. This benchmark economy can be viewed as an economy in which the exchange is required to set the cost, q, of observing the market price to 0. In this case all traders have price information; that is, $\mu = 1$.

The equilibrium price of the risky asset $\tilde{p}^F$ is the equilibrium price in Proposition 1 with

$\mu = 1$. The welfare of each rational trader is:

$$WEL_R^F = -\frac{1}{\gamma}\log\left(-E\left[VI\left(\tilde{p}^F, \tilde{s}_i\right)\right]\right)$$

where

$$E\left[VI\left(\tilde{p}^F, \tilde{s}_i\right)\right] = -\sqrt{\frac{Var\left(\tilde{v}-\tilde{p}^F|\tilde{p}^F, \tilde{s}_i\right)}{Var\left(\tilde{v}-\tilde{p}^F\right)}}\exp\left\{-\frac{\left[E\left(\tilde{v}-\tilde{p}^F\right)\right]^2}{2Var\left(\tilde{v}-\tilde{p}^F\right)}\right\}.$$

The revenue that a liquidity trader receives from selling $\tilde{x}$ is $\left(\tilde{p}^F - \tilde{v}\right)\tilde{x}$. The expected revenue $E\left[\left(\tilde{p}^F - \tilde{v}\right)\tilde{x}\right]$ is thus the negative of the expected opportunity cost $E\left[\left(\tilde{v} - \tilde{p}^F\right)\tilde{x}\right]$ associated with a trade of $\tilde{x}$ shares. We use expected revenue to proxy for the welfare of liquidity traders to capture the idea that they prefer to realize their unmodeled hedging or liquidity needs at the smallest possible expected opportunity cost.[13] Thus we let

$$\begin{aligned} WEL_L^F &\equiv E\left[\left(\tilde{p}^F - \tilde{v}\right)\tilde{x}\right] \\ &= -\bar{x}E\left(\tilde{v}-\tilde{p}^F\right) - \lambda^F Var\left(\tilde{x}\right) \end{aligned}$$

where the second equality follows from price function (3) and immediately implies that the welfare of liquidity traders decreases in the cost of capital and increases in market liquidity. By default, in this economy the exchange has a profit of $\pi^F = 0$.

B. Economy D versus Economy F

We label the economy analyzed in the previous sections as "Economy D". We denote the equilibrium price of the risky asset resulting from the exchange's optimal price of information, q^*, by $\tilde{p}^*$. This price is obtained by setting $\mu = \mu^*$ in Proposition 1. The profit maximizing

[13]This is not explicitly derived from expected utility as we do not have a utility function for liquidity traders. Instead, it is the expected revenue from their trade.

exchange charges a price q^* so that each trader is indifferent between becoming price-informed and paying q^* and remaining price-uninformed. Thus the welfare of each rational trader is:

$$WEL_R^D = -\frac{1}{\gamma}\log\left(-E\left[VU\left(\tilde{s}_i\right)\right]\right)$$

where

$$E\left[VU\left(\tilde{s}_i\right)\right] = -\sqrt{\frac{Var\left(\tilde{v}-\tilde{p}^*|\tilde{s}_i\right)}{Var\left(\tilde{v}-\tilde{p}^*\right)}}\exp\left\{-\frac{\left[E\left(\tilde{v}-\tilde{p}^*\right)\right]^2}{2Var\left(\tilde{v}-\tilde{p}^*\right)}\right\}.$$

In this economy the welfare of liquidity traders is:

$$WEL_L^D \equiv E\left[\left(\tilde{p}^*-\tilde{v}\right)\tilde{x}\right] = -\bar{x}E\left(\tilde{v}-\tilde{p}^*\right) - \lambda^* Var\left(\tilde{x}\right)$$

and the exchange has a profit of $\pi^D = \mu^* B\left(\mu^*\right)$. Combining Corollaries 1-4 and using the expressions of WEL_L^F and WEL_L^D, we have the following proposition.

Proposition 3 *Price informativeness is lower and the cost of capital and return volatility are higher in Economy D than in Economy F. If the risk aversion parameter γ is large enough, liquidity is lower and liquidity traders are worse-off in Economy D than in Economy F.*

Thus allowing the exchange to sell price information at the profit maximizing price reduces price informativeness and increases volatility. Under reasonable specifications of the economy it also lowers liquidity and harms liquidity traders.

Table II illustrates the comparison between Economy D and Economy F for the parameter configuration given by Table I.

[INSERT TABLE II HERE]

If the exchange is allowed to sell access to price information, we move from Economy F to Economy D, and liquidity traders lose while the exchange wins. For rational traders, the welfare effect is ambiguous, because of two competing effects: (i) Economy D is less transparent and rational traders can make more profit from trading with liquidity traders; and, (ii) In economy D rational traders have to pay a cost of q^* to purchase the price data, while in Economy F it is free. For the numerical example in Table II, the positive effect dominates so that rational traders are better-off in Economy D. The total welfare of all market participants, defined as the sum of the welfare of all traders and the exchange's profit, is however lower in Economy D.[14] The positive implications in Table II are also consistent with Proposition 3—market efficiency and liquidity decrease and the cost of capital and return volatility increase if the exchange is allowed to sell price information.

V. Endogenous Information Acquisition

In this section we extend our analysis to allow rational traders to decide whether to acquire private information in addition to deciding whether to acquire price information. Now traders are not endowed with signals. They can purchase neither, one or both of the signals, $\tilde{p}$ and $\tilde{s}_i$, at costs $q > 0$ and $c > 0$, respectively. The analysis in this section will thus capture the impact of selling price information on the total amount of fundamental information.

There are two information purchase decisions so there are potentially four combinations of information purchase decisions and thus four types of traders. We denote them as PS-informed (of mass μ_{ps}, with information $\{\tilde{p}, \tilde{s}_i\}$), S-informed (of mass μ_s, with information

[14]We can also compare Economy D to an economy in which no one has price information. This is an approximation to case of Europe where no consolidated tape exists. There is no equilibrium in our model with $\mu = 0$ as in this case no traders demand would depend on the price. But we can take the limit of the equilibria and welfares as $\mu \to 0$. Every trader is worse-off in this no information economy than they are in either Economy F or Economy D.

$\{\tilde{s}_i\}$), P-informed (of mass μ_p, with information $\{\tilde{p}\}$), and Uninformed (of mass μ_u, without any information). We will show that all four groups cannot coexist. Specifically, for intermediate values of c no one purchases fundamental information without also purchasing price information, i.e. there are no S-informed traders.

A. Financial Market Equilibrium

There is a rational expectations equilibrium in which the price function still has the form

$$\tilde{p} = \alpha + \beta\tilde{v} - \lambda\tilde{x},$$

and observing price is equivalent to observing the signal

$$\tilde{s}_p = \tilde{v} - \frac{\lambda}{\beta}\left(\tilde{x} - \bar{x}\right) = \tilde{v} - m^{-1}\left(\tilde{x} - \bar{x}\right).$$

Using computations similar to those in the previous sections it is easy to show that the demand functions are

$$\begin{aligned}
D_{ps}\left(\tilde{p};\tilde{s}_i\right) &= \frac{E\left(\tilde{v}|\tilde{p},\tilde{s}_i\right) - \tilde{p}}{\gamma Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)} = \frac{\rho_v\bar{v} + \rho_\varepsilon\tilde{s}_i + m^2\rho_x\tilde{s}_p - \left(\rho_v + \rho_\varepsilon + m^2\rho_x\right)\tilde{p}}{\gamma} \\
D_s\left(\tilde{s}_i\right) &= \frac{E\left(\tilde{v} - \tilde{p}|\tilde{s}_i\right)}{\gamma Var\left(\tilde{v} - \tilde{p}|\tilde{s}_i\right)} = \left(\frac{1}{\gamma}\right)\frac{-\alpha + \left(1-\beta\right)\frac{\rho_v\bar{v}+\rho_\varepsilon\tilde{s}_i}{\rho_v+\rho_\varepsilon} + \lambda\bar{x}}{\left(1-\beta\right)^2\frac{1}{\rho_v+\rho_\varepsilon} + \lambda^2\frac{1}{\rho_x}}, \\
D_p\left(\tilde{p}\right) &= \frac{E\left(\tilde{v}|\tilde{p}\right) - \tilde{p}}{\gamma Var\left(\tilde{v}|\tilde{p}\right)} = \frac{\rho_v\bar{v} + m^2\rho_x\tilde{s}_p - \left(\rho_v + m^2\rho_x\right)\tilde{p}}{\gamma}, \\
D_u &= \frac{E\left(\tilde{v} - \tilde{p}\right)}{\gamma Var\left(\tilde{v} - \tilde{p}\right)} = \left(\frac{1}{\gamma}\right)\frac{-\alpha + \left(1-\beta\right)\bar{v} + \lambda\bar{x}}{\left(1-\beta\right)^2\frac{1}{\rho_v} + \lambda^2\frac{1}{\rho_x}}.
\end{aligned}$$

The market clearing condition is

$$\int_0^{\mu_{ps}} D_{ps}\left(\tilde{p};\tilde{s}_i\right)di+\int_0^{\mu_s} D_s\left(\tilde{s}_i\right)di+\mu_p D_p\left(\tilde{p}\right)+\mu_u D_u=\tilde{x}.$$

Solving the market clearing condition for $\tilde{p}$ yields the following proposition which generalizes Proposition 1 to an economy with all four types of traders.

Proposition 4 *In the economy with endogenous private information acquisition, there exists a partially revealing REE, in which the price function is*

$$\tilde{p}=\alpha+\beta\tilde{v}-\lambda\tilde{x},$$

where

$$\beta = \frac{\gamma m+\left(\mu_{ps}+\mu_p\right)m^2\rho_x}{\mu_{ps}\rho_\varepsilon+\left(\mu_{ps}+\mu_p\right)\left(\rho_v+m^2\rho_x\right)}, \tag{25}$$

$$\lambda = \frac{\gamma+\left(\mu_{ps}+\mu_p\right)m\rho_x}{\mu_{ps}\rho_\varepsilon+\left(\mu_{ps}+\mu_p\right)\left(\rho_v+m^2\rho_x\right)}, \tag{26}$$

$$\alpha=\left(1-\beta\right)\bar{v}+\lambda\bar{x}-\frac{\gamma\bar{x}}{\mu_{ps}Var^{-1}\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)+\mu_s Var^{-1}\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)+\mu_p Var^{-1}\left(\tilde{v}-\tilde{p}|\tilde{p}\right)+\mu_u Var^{-1}\left(\tilde{v}-\tilde{p}\right)}, \tag{27}$$

where m is determined by equation

$$\mu_{ps}+\mu_s\frac{\left[\mu_{ps}\rho_\varepsilon+\left(\mu_{ps}+\mu_p\right)\rho_v-\gamma m\right]\left[\mu_{ps}\rho_\varepsilon+\left(\mu_{ps}+\mu_p\right)\left(\rho_v+m^2\rho_x\right)\right]}{\left[\mu_{ps}\rho_\varepsilon+\left(\mu_{ps}+\mu_p\right)\rho_v-\gamma m\right]^2+\left[\gamma+\left(\mu_{ps}+\mu_p\right)m\rho_x\right]^2\frac{\rho_v+\rho_\varepsilon}{\rho_x}}-\frac{\gamma m}{\rho_\varepsilon}=0 \tag{28}$$

and where

$$Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)=\frac{1}{\rho_v+\rho_\varepsilon+m^2\rho_x}, \tag{29}$$

$$Var(\tilde{v}-\tilde{p}|\tilde{s}_i) = (1-\beta)^2 \frac{1}{\rho_v+\rho_\varepsilon} + \lambda^2 \frac{1}{\rho_x}, \tag{30}$$

$$Var(\tilde{v}-\tilde{p}|\tilde{p}) = \frac{1}{\rho_v + m^2\rho_x}, \tag{31}$$

$$Var(\tilde{v}-\tilde{p}) = (1-\beta)^2 \frac{1}{\rho_v} + \lambda^2 \frac{1}{\rho_x}. \tag{32}$$

Applying equation (42) from Appendix A, we can show that the ex ante utilities of PS-informed, S-informed, P-informed and uninformed traders are as follows:

$$E[V_{ps}(\tilde{p},\tilde{s}_i)] = \sqrt{\frac{Var(\tilde{v}-\tilde{p}|p,\tilde{s}_i)}{Var(\tilde{v}-\tilde{p})}} e^{\gamma(c+q)} E[V_u], \tag{33}$$

$$E[V_s(\tilde{s}_i)] = \sqrt{\frac{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}{Var(\tilde{v}-\tilde{p})}} e^{\gamma c} E[V_u], \tag{34}$$

$$E[V_p(\tilde{p})] = \sqrt{\frac{Var(\tilde{v}-\tilde{p}|\tilde{p})}{Var(\tilde{v}-\tilde{p})}} e^{\gamma q} E[V_u], \tag{35}$$

$$E[V_u] = -\exp\left\{-\frac{[E(\tilde{v}-\tilde{p})]^2}{2Var(\tilde{v}-\tilde{p})}\right\}. \tag{36}$$

Accordingly, their certainty equivalents are respectively $-\frac{1}{\gamma}\log(-E[V_{ps}(\tilde{p},\tilde{s}_i)])$, $-\frac{1}{\gamma}\log(-E[V_s(\tilde{s}_i)])$, $-\frac{1}{\gamma}\log(-E[V_p(\tilde{p})])$, and $-\frac{1}{\gamma}\log(-E[V_u])$.

Equation (28) only implicitly determines the price informativeness, m. However, we can solve for m analytically in the cases that are most important for our analysis. First, suppose that there are no traders who purchase price information and do not purchase fundamental information, $\mu_p = 0$. In this case, $m = \frac{\mu_{ps}\rho_\varepsilon\left(\gamma^2+\mu_{ps}^2\rho_v\rho_x+\mu_{ps}^2\rho_\varepsilon\rho_x+\mu_{ps}\mu_s\rho_v\rho_x+\mu_{ps}\mu_s\rho_\varepsilon\rho_x\right)}{\gamma\left(\gamma^2+\mu_{ps}^2\rho_v\rho_x+\mu_{ps}^2\rho_\varepsilon\rho_x+\mu_{ps}\mu_s\rho_\varepsilon\rho_x\right)}$, which is a generalization of the expression for m in Proposition 1. This result is useful when the information acquisition cost c is low so that the economy is close to our baseline model. Second, suppose that there are no traders who purchase fundamental information but not price information, $\mu_s = 0$. We will show shortly that this is the dominant case when the information acquisition cost c is intermediate. This case will be the primary focus of our analysis as it seems the most relevant (some traders are informed and others are not). The following corollary provides the solution for m in this case.

Corollary 5 *When $\mu_s = 0$, we have $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$.*

B. Economy F

We first describe equilibrium information acquisition decisions in the benchmark economy, Economy F, in which all traders observe price information for free. In this economy, traders are either PS-informed or P-informed; that is, $\mu_p + \mu_{ps} = 1$.

By Corollary 5, we have

$$m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}.$$

Equations (29), (31), (33) and (35), imply that the benefit of observing signal $\tilde{s}_i$ is

$$B^F\left(\mu_{ps}\right) \equiv \frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}\right] = \frac{1}{2\gamma}\log\left[1+\frac{\rho_\varepsilon}{\rho_v+\left(\frac{\mu_{ps}\rho_\varepsilon}{\gamma}\right)^2\rho_x}\right], \tag{37}$$

which is decreasing in μ_{ps}.

It will be useful to define two extreme values for the benefit of acquiring private signals. This benefit is largest when no one else is informed, $\mu_{ps} = 0$, and it is smallest when everyone else is informed, $\mu_{ps} = 1$. Specifically, let us define

$$\bar{c} \equiv B^F(0) = \frac{1}{2\gamma}\log\left(1+\frac{\rho_\varepsilon}{\rho_v}\right), \tag{38}$$

$$\underline{c} \equiv B^F(1) = \frac{1}{2\gamma}\log\left(1+\frac{\rho_\varepsilon}{\rho_v+\left(\frac{\rho_\varepsilon}{\gamma}\right)^2\rho_x}\right). \tag{39}$$

If the cost of acquiring signals is too large, $c \geq \bar{c}$, then the equilibrium fraction μ_{ps}^F of PS-informed traders is equal to 0. Alternatively, if it is sufficiently low, $c \leq \underline{c}$, then $\mu_{ps}^F = 1$. Finally, if $c \in (\underline{c}, \bar{c})$, μ_{ps}^F is determined by $B^F\left(\mu_{ps}\right) = c$, which is formally characterized by the following proposition.

Proposition 5 *Suppose $\underline{c} \leq c \leq \bar{c}$. The equilibrium fraction μ_{ps}^F of PS-informed traders in Economy F is*

$$\mu_{ps}^F = \frac{\gamma}{\rho_\varepsilon}\sqrt{\frac{\frac{\rho_\varepsilon}{e^{2\gamma c}-1} - \rho_v}{\rho_x}},$$

and the price informativeness is $m^F = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$.

When $c \leq \underline{c}$, all traders will acquire the private signal $\tilde{s}_i$ in Economy F. If c is sufficiently small, most traders will also acquire the signal $\tilde{s}_i$ in Economy D. When $c \geq \bar{c}$, no traders will acquire signal $\tilde{s}_i$ in Economy F. This is also the case in Economy D. For the remainder of this section, we will focus on the interesting case in which $\underline{c} \leq c \leq \bar{c}$. We next illustrate how selling price information affects information acquisition and equilibrium outcomes for these intermediate values of the cost of acquiring private information.

C. Economy D

In Appendix D, we use Lemmas 1-3 to characterize the demand function for price information in the economy in which traders also chose whether to acquire private information. This demand function is continuous, decreasing, and its behavior as a function of the cost of acquiring price information, q, can be described in three ranges as follows.

Range 1. When $q \in [0, q_1]$, all traders purchase price information, and there is a μ_{ps}^F fraction of PS-informed traders and $\left(1 - \mu_{ps}^F\right)$ fraction of P-informed traders. The critical constant q_1 is given by equation (46) in Appendix D.

Range 2. When $q \in (q_1, q_2)$, the demand for price data is $\mu_{ps}^F + \mu_p$, where μ_p is determined by $B_M(\mu_p) = q$. There is a μ_{ps}^F fraction of PS-informed traders, a μ_p fraction of P-informed traders and $(1 - \mu_{ps}^F - \mu_p)$ fraction of uninformed traders. The critical constant q_2 is given by equation (50) and the function $B_M(\cdot)$ is given by equation (47) in Appendix D.

Range 3. When $q \in [q_2, \infty)$, the demand for the price data is μ_{ps}, where μ_{ps} is determined by $B_H(\mu_{ps}) = q + c$. There is a μ_{ps} fraction of PS-informed traders and $(1 - \mu_{ps})$ fraction of uninformed traders. The function $B_H(\cdot)$ is given by equation (53) in Appendix D.

Note that when c falls into the intermediate range of $[\underline{c}, \bar{c}]$, there are no S-informed traders. The following proposition, which shows that the marginal benefit of acquiring an additional form of information is greater for traders who have already obtained some information than for those who have not, provides intuition for this fact.

Proposition 6 ***[Learning Complementarities]*** *Suppose $\mu_s = 0$. The extra benefit of observing price information $\tilde{p}$ is higher for S-informed traders (who have already purchased fundamental information $\tilde{s}_i$) than for uninformed traders. Similarly, the benefit of observing fundamental information $\tilde{s}_i$ is higher for P-informed traders (who have already purchased price information $\tilde{p}$) than for uninformed traders.*

Proposition 6 implies that the two pieces of information $\tilde{p}$ and $\tilde{s}_i$ are complementary. That is, traders who have purchased one signal, have a greater incentive to acquire another signal than those who haven't purchased any signal. This occurs because the signals $\tilde{s}_i$ and $\tilde{p}$ are useful for predicting two non-perfectly correlated components $\tilde{v}$ and $\tilde{p}$, respectively, of the uncertain profit per unit of the asset $(\tilde{v} - \tilde{p})$.

Boulatov and Dierker (2007) provide a related result showing that price data is more valuable to those traders with more precise private fundamental information. Our analysis differs from theirs in two important ways. First, the precision of price information in their model is exogenous. They employ a standard Kyle model with risk neutral traders and market makers. The price is therefore set by market makers who receive an *exogenous* new piece of dividend information which is incorporated into prices that will be sold by the exchange. Second, they don't allow traders to acquire fundamental information endogenously. We believe these two issues are essential features of price information, because they are responsible for the feedback effects that are unique to price information.

Using these results we can now analyze the demand function for price information. Intuitively, when the exchange sets $q = 0$, so that the economy is described by Economy F, all rational traders choose to observe p, and there are only two types of traders: PS-informed and P-informed with the fraction of PS-informed traders given by μ_{ps}^{F}. Similarly, when q is low, only PS- and P- informed traders are active, and this pattern will persist until q reaches the threshold value of q_1.

Suppose the exchange increases q above q_1. By Proposition 6, we know that the benefit that P-informed traders receive from price information is smaller than the benefit of price information for PS-informed traders. So as we increase q the first traders who would chose to not buy price information are P-informed traders. The exchange will charge a price q to make traders indifferent between being P-informed and being uninformed. As q increases, more P-informed traders will become uninformed, and when q increases to a threshold value of q_2, all P-informed traders switch to becoming uninformed traders. Note that during this process, the fraction of traders observing fundamental information $\tilde{s}_i$ is unchanged at μ_{ps}^{F}.

Once the price q reaches q_2, all P-informed traders switch to being uninformed. As the exchange continues to raise q, some PS-informed traders will not purchase the price data as well. Recall that being informed of $\tilde{s}_i$ only is not worth the cost c of acquiring it. So, those PS-informed traders who switch types will choose to become uninformed traders, that is, they give up two pieces of information simultaneously. As a result, only two types of traders, PS-informed and uninformed traders, are active in the market. This result is driven by the complementarity effect described by Proposition 6: Being informed of any one piece of information ($\tilde{p}$ or $\tilde{s}_i$) is not worth the separate cost, but being informed of two pieces of information ($\tilde{p}$ and $\tilde{s}_i$) generates a complement effect, which strengthens the benefit. As a consequence, traders either choose to become informed of both pieces of information simultaneously, or to stay uninformed.

Now that we have the demand function for price information, we can determine the price of price data that maximizes the exchange's profit. From the previous discussion, we also know that if the optimal price q^* is set so that the exchange ends up in region 3, that is, the demand is equal to $\mu_{ps}^* < \mu_{ps}^F$, then relative to Economy F where price data is free, there is less fundamental information in Economy D. Formally, we have the following proposition.

Proposition 7 ***[Crowding-Out Effect]*** *Suppose $\underline{c} \leq c \leq \bar{c}$ and $\gamma \geq \frac{1}{2}\sqrt{\frac{\rho_v \rho_\varepsilon \rho_x}{\rho_v + \rho_\varepsilon}}$. If the optimal price q^* set by the exchange is greater than q_2, then there is less fundamental information in Economy D than in Economy F.*

The condition $\gamma \geq \frac{1}{2}\sqrt{\frac{\rho_v \rho_\varepsilon \rho_x}{\rho_v + \rho_\varepsilon}}$ ensures that in Region 3 uninformed traders do not want to deviate to become S-informed, so that the demand function for the price data is consistent with equilibrium. Actually, this range of risk aversions is quite large. For example, with the parameter configuration in Table I, we have $\frac{1}{2}\sqrt{\frac{\rho_v \rho_\varepsilon \rho_x}{\rho_v + \rho_\varepsilon}} = 2.04$, which can be easily satisfied. Figure 3 plots the demand function (Panel (a)) and the profit function (Panel (b)) of the exchange using the parameter configuration given by Table I. The cost c of acquiring the fundamental information $\tilde{s}_i$ is set at 0.0445. The corresponding total amount of fundamental information μ_{ps}^F is 0.1635. The optimal price is $q^* = 0.4064 > q_2 = 0.1725$, and the resulting optimal demand for the price data is $\mu_{ps}^* = 0.0822$. The high price of the price data in Economy D crowds out almost half of fundamental information relative to Economy F (i.e., $\frac{\mu_{ps}^F - \mu_{ps}^*}{\mu_{ps}^F} \approx \frac{1}{2}$).

[INSERT FIGURE 3 HERE]

In addition, in Appendix D where we derive the demand function for price data, we also use Lemmas 1-3 to show that as the exchange increases price q, the positive variables generally become worse. Given that Economy F corresponds to an economy with $q = 0$, we have the following proposition, which implies that our Proposition 3 is robust to endogenous fundamental information acquisition. The proof is simply a combinations of Lemmas 1-3 in Appendix D.

Proposition 8 ***[Economy D versus Economy F]*** *Suppose* $\underline{c} \le c \le \bar{c}$ *and* $\gamma \ge \frac{1}{2}\sqrt{\frac{\rho_v \rho_\varepsilon \rho_x}{\rho_v + \rho_\varepsilon}}$*. Relative to Economy F, Economy D has a (weakly) lower price informativeness, a higher cost of capital, and a higher return volatility. When* γ *is large, or* ρ_v *(or* ρ_ε*) is small, Economy D also has a lower liquidity, and liquidity traders become worse-off.*

Figure 4 plots the equilibrium outcomes in Economy D and Economy F against the cost c of acquiring fundamental information $\tilde{s}_i$, when all other parameters are set at values given by Table I. We can see that for all values of c in $[\underline{c}, \bar{c}]$, there is a crowding-out effect on fundamental information production in Economy D (Panel (a)), and all positive variables become worse (Panels (b)-(e)). Rational traders are better-off in Economy D than in Economy F (Panel (f)), liquidity traders are worse-off (Panel (g)), and the total welfare is lower (Panel (h)).

[INSERT FIGURE 4 HERE]

VI. Mine versus Ours: Tape Data and Regulatory Policy

Our analysis shows that allowing exchanges to sell price data to traders can introduce important price discovery, cost of capital, return volatility, liquidity, and welfare effects. These effects arise because trade information is valuable, both to the traders who know it and to the exchanges who produce it. It is hardly surprising, therefore, that exchanges want to sell trade data and some traders want to buy it. The question of interest, however, is what should be the regulatory policy regarding the access to and distribution of trading process information?

To understand the context for this debate, it is useful to consider the current situation in both the U.S. and Europe with respect to tape data. As noted earlier, the US has a consolidated tape to which trades and quotes, in principle, must be reported in real time. In practice, some trades (odd lots, for example) are not reported to the tape, and the processing of the trades and quotes that are reported requires some time. Whereas in times past these delays ran to several seconds, in 2008 the average latency of the tape was on the order of 20 -50 milliseconds, and it is now variously estimated at between 5 and 10 milliseconds.[15] In Europe, there is no mandatory consolidated tape (MCT), and exchanges and trading platforms sell proprietary data feeds. Most US exchanges and markets also provide trade data products, and selling information is a significant source of profit for exchanges worldwide.

The Securities and Exchange Commission (SEC) and the Committee of European Securities Regulators (CESR) are both currently reviewing their respective market structures

[15]Reg NMS did not require exchanges to stop providing data feeds but it did require that they send data to its subscribers at the same time that they send it to the consolidated tape. The current latency difference is thus determined by the time it takes to process the data at the consolidated level. For estimates of current latency speeds see BATS comment letter April 10, 2010.

with respect to data issues. In Europe, the question under review is whether to establish a mandatory consolidated tape. The focus here is on the role price information plays "in achieving efficient price discovery and facilitating the achievement and monitoring of best execution."[16] In the US, the issue is proprietary data feeds, with the SEC posing the issue as being one of "fairness". Specifically, the SEC asks "is the existence of any latency, or disparity in information transmitted, fair to investors or other market participants that rely on the consolidated market data feeds and do not use individual trading center data feeds?"[17]

Both issues essentially involve questions of differential access to price information, and our analysis provides insights into the debate. Turning first to the overall issue of a mandatory tape, our analysis shows that market efficiency (i.e. price discovery) and market liquidity are higher and the cost of capital and return volatility are lower when all traders observe prices for "free" than when traders have to pay a cost to purchase price data from exchanges. When traders all have price information, the adverse selection problems that arise with differential access are mitigated and this reduces the risk premium that otherwise uninformed traders would require to participate in the market. In addition, our analysis shows that observing price information encourages the production of fundamental information, which further improves price discovery. In general, market efficiency is enhanced in equilibrium with greater access to price information and this would appear to be most easily attained with a consolidated tape. Our findings here are thus opposite of those of Cespa and Foucault (2008), who argue that the optimal market structure would be more opaque with respect to price information.

Whether exchanges should also be allowed to sell proprietary data feeds is more complex.

[16]See CESR Technical Advice to the European Commission in the Context of the MiFID Review and Responses to the European Commission Request for Additional Information, July 2010, page 28.

[17]See SEC Concept Release on Equity Market Structure, April 2010, page 62.

Exchanges defend the practice of selling data by noting that such data are not costless to produce and it allows them to invest in the costly trading systems needed to produce high quality trades and quotes. While the exchanges share in the tape revenues from the consolidated tape, selling proprietary data allows trading venues to better meet the needs of specialized trading groups. BATS (the 3rd largest trading venue in the U.S.), for example, actually gives away its data feeds as a competitive inducement to attract high frequency traders to its trading platform. The exchanges further note that access to the data is "fair" in that they are willing to sell data to any or all traders willing to pay for it.

We believe that the SEC's query regarding fairness misses the bigger picture. Selling data in our model does result in some traders doing better than others, and it particularly benefits exchanges.[18] Yet, these redistributive effects are only part of the story. Allowing some traders to purchase better information affects price discovery, the cost of capital, return volatility and market liquidity. It is these latter effects that we believe should be the focus of regulatory concern. Fairness, per se, is not necessarily a good goal for market design because market participants are not all inherently equal.[19] In our view, allowing exchanges to sell price information is undesirable because it reduces efficiency and market quality, and the practice should be restricted.

Should the SEC also preclude exchanges from providing other data that is not part of the price and quote montage? Unlike prices which are fundamental information for all traders, specialized information is more likely to be valuable to traders pursuing particular trading strategies. Selling data not in the quote and price montage can potentially lower trading costs for those traders and would therefore seem an acceptable practice. What is interesting

[18]Note, however, that rational traders who purchase price information pay exactly what it is worth to them and have the same expected utility as rational, price-uninformed traders.

[19]The SEC has not traditionally required a "one size fits all" market structure in recognition of the needs that different traders face.

to contemplate, however, is that providing information to some traders can have unexpected effects. As Easley and O'Hara (2010) demonstrate, ambiguity can reduce participation in markets. To the extent that traders perceive greater ambiguity attaching to markets which selectively sell data, they can opt to trade elsewhere or not at all.[20] Such an outcome will surely restrict an exchange's data sales without regulatory involvement.

[20]Such a situation was alleged to have occurred in May 2010 in Europe with respect to data distributed by Chi-X and BATS Europe. In a report from Themis Trading Arnuk and Saluzzi (2010) alleged that high speed data packages sold by BATS and NASDAQ-OMX allowed purchasers to discern the existence of hidden orders, thereby disadvantaging hidden limit order traders. Disclosure of this practice prompted European trading platform Turquoise to issue a statement to the effect that its data feeds did not reveal such information. Traders subsequently routed order flow away from Chi-X and BATS, causing a dramatic fall in both venues trading volume. Chi-X and BATS subsequently changed their data feeds to limit the data revealed.

Appendix

A. A Technical Note

This appendix provides the derivation of the demand function and indirect utility function (and its expectations with respect to a smaller information set) for the CARA-normal setup. Let $\mathcal{I}_i$ be the information set of trader i, where $\mathcal{I}_i$ can include $\tilde{p}$ or not. The utility maximization problem of a CARA-trader is

$$\max_D E\left[-e^{-\gamma \tilde{W}_i} \middle| \mathcal{I}_i\right]$$

subject to:

$$\tilde{W}_i = \bar{W}_{i0} + D\left(\tilde{v} - \tilde{p}\right)$$

where $\bar{W}_{i0}$ is a constant since investors are endowed with only risk-free assets.

The expected utility maximizing demand is

$$D^* = \frac{E\left(\tilde{v} - \tilde{p} | \mathcal{I}_i\right)}{\gamma Var\left(\tilde{v} - \tilde{p} | \mathcal{I}_i\right)} \tag{40}$$

and the indirect utility function is

$$V\left(\mathcal{I}_i\right) = -\exp\left\{-\gamma \bar{W}_{i0} - \frac{\left[E\left(\tilde{v} - \tilde{p} | \mathcal{I}_i\right)\right]^2}{2Var\left(\tilde{v} - \tilde{p} | \mathcal{I}_i\right)}\right\}. \tag{41}$$

Now suppose we want to condition some coarser information set. Let $\mathcal{I}_{\text{small}}$ be an information set that is coarser than $\mathcal{I}_i$. Then, following the derivation of Grossman-Stiglitz (1980) and using the moment generating function of noncentral chi-square distributions, we have

$$E\left[V\left(\mathcal{I}_i\right)|\mathcal{I}_{\text{small}}\right]=-\sqrt{\frac{Var\left(\tilde{v}-\tilde{p}|\mathcal{I}_i\right)}{Var\left(\tilde{v}-\tilde{p}|\mathcal{I}_{\text{small}}\right)}}\exp\left[-\gamma\bar{W}_{i0}-\frac{\left[E\left(\tilde{v}-\tilde{p}|\mathcal{I}_{\text{small}}\right)\right]^2}{2Var\left(\tilde{v}-\tilde{p}|\mathcal{I}_{\text{small}}\right)}\right]. \tag{42}$$

B. Proof of Corollaries 2-4

***Proof of Corollary* 2** Direct computation shows

$$\begin{aligned}&\frac{\partial}{\partial\mu}\left(\frac{\mu}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)}+\frac{1-\mu}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}\right)\\ =\ &\left[\frac{1}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)}-\frac{1}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}\right]+\left[\mu\frac{\partial}{\partial\mu}\frac{1}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)}+\left(1-\mu\right)\frac{\partial}{\partial\mu}\frac{1}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}\right].\end{aligned}$$

We next establish that both bracketed terms are positive, which is a sufficient condition for $\frac{\partial CC}{\partial\mu}<0$ by equation (22).

Clearly, $\frac{1}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}<\frac{1}{Var(\tilde{v}|\tilde{p},\tilde{s}_i)}$, and as a result, $\left[\frac{1}{Var(\tilde{v}|\tilde{p},\tilde{s}_i)}-\frac{1}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]>0$.

By equation (20) and by $\frac{\partial m}{\partial\mu}>0$, we know $\frac{\partial}{\partial\mu}\frac{1}{Var(\tilde{v}|\tilde{p},\tilde{s}_i)}>0$. By equation (19),

$$\frac{1}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}=\frac{\mu^2\rho_x\left(\rho_v+\rho_\varepsilon\right)}{\gamma^2+\mu^2\rho_x\left(\rho_v+\rho_\varepsilon\right)}\frac{1}{Var\left(\tilde{v}|\tilde{p},\tilde{s}_i\right)}.$$

So, $\frac{1}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}$ is increasing in μ as well, since both $\frac{\mu^2\rho_x(\rho_v+\rho_\varepsilon)}{\gamma^2+\mu^2\rho_x(\rho_v+\rho_\varepsilon)}$ and $\frac{1}{Var(\tilde{v}|\tilde{p},\tilde{s}_i)}$ are increasing in μ; that is, $\frac{\partial}{\partial\mu}\frac{1}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}>0$. Thus, $\left[\mu\frac{\partial}{\partial\mu}\frac{1}{Var(\tilde{v}|\tilde{p},\tilde{s}_i)}+\left(1-\mu\right)\frac{\partial}{\partial\mu}\frac{1}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]>0$.

***Proof of Corollary* 3** By the variance decomposition formula,

$$Var\left(\tilde{v}-\tilde{p}\right)=Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)+Var\left[E\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)\right].$$

We have shown

$$\frac{\partial Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}{\partial \mu}<0.$$

Now examine $\frac{\partial Var[E(\tilde{v}-\tilde{p}|\tilde{s}_i)]}{\partial \mu}$. By equation (12),

$$Var\left[E\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)\right]=\left(1-\beta\right)^2\frac{\rho_\varepsilon}{\left(\rho_v+\rho_\varepsilon\right)\rho_v}.$$

By equation (16) in Proposition 1,

$$\left(1-\beta\right)=\frac{\rho_v+\rho_\varepsilon-\left(\gamma/\mu\right)m}{\rho_v+\rho_\varepsilon+m^2\rho_x},$$

and the definition of m in Proposition 1 shows:

$$\rho_v+\rho_\varepsilon-\left(\gamma/\mu\right)m=\rho_v\frac{\gamma^2+\mu^2\rho_v\rho_x+\mu^2\rho_x\rho_\varepsilon}{\gamma^2+\mu^2\rho_v\rho_x+\mu\rho_x\rho_\varepsilon}>0.$$

Therefore, $(1-\beta)>0$.

Direct computation shows:

$$\frac{\partial\left(1-\beta\right)}{\partial m}=-\frac{\left(\gamma/\mu\right)\left(\rho_v+\rho_\varepsilon+m^2\rho_x\right)+\left(\rho_v+\rho_\varepsilon-\left(\gamma/\mu\right)m\right)\left(2m\rho_x\right)}{\left(\rho_v+\rho_\varepsilon+m^2\rho_x\right)^2}<0,$$

because $\left(\gamma/\mu\right)\left(\rho_v+\rho_\varepsilon+m^2\rho_x\right)>0$ and $\left(\rho_v+\rho_\varepsilon-\left(\gamma/\mu\right)m\right)>0$.

Thus,

$$\frac{\partial Var\left[E\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)\right]}{\partial \mu}=2\left(1-\beta\right)\frac{\rho_\varepsilon}{\left(\rho_v+\rho_\varepsilon\right)\rho_v}\frac{\partial\left(1-\beta\right)}{\partial m}\frac{\partial m}{\partial \mu}<0$$

and hence

$$\frac{\partial Var\left(\tilde{v}-\tilde{p}\right)}{\partial \mu}=\frac{\partial Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}{\partial \mu}+\frac{\partial Var\left[E\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)\right]}{\partial \mu}<0.$$

***Proof of Corollary* 4** Direct computation shows:

$$\frac{\partial\left(1/\lambda\right)}{\partial\mu}=\frac{\left[2m\left(\gamma/\mu\right)+m^{2}\rho_{x}-\left(\rho_{v}+\rho_{\varepsilon}\right)\right]\rho_{x}\frac{\partial m}{\partial\mu}+\frac{\gamma}{\mu^{2}}\left(\rho_{v}+\rho_{\varepsilon}+m^{2}\rho_{x}\right)}{\left(\left(\gamma/\mu\right)+m\rho_{x}\right)^{2}}. \tag{43}$$

Then, we can show $\frac{\partial(1/\lambda)}{\partial\mu}>0$ if one of the following three conditions holds:

(1) γ is sufficiently large. By equations (18) and (21),$\lim_{\gamma\to 0}m=\lim_{\gamma\to 0}\frac{\partial m}{\partial\mu}=0$ and hence the first term in the numerator of equation (43), $\left[2m\left(\gamma/\mu\right)+m^{2}\rho_{x}-\left(\rho_{v}+\rho_{\varepsilon}\right)\right]\rho_{x}\frac{\partial m}{\partial\mu}=\left[2\rho_{\varepsilon}\frac{\left(\gamma^{2}+\mu\rho_{v}\rho_{x}+\mu\rho_{x}\rho_{\varepsilon}\right)}{\left(\gamma^{2}+\mu^{2}\rho_{v}\rho_{x}+\mu\rho_{x}\rho_{\varepsilon}\right)}+m^{2}\rho_{x}-\left(\rho_{v}+\rho_{\varepsilon}\right)\right]\rho_{x}\frac{\partial m}{\partial\mu}\to 0$, but the second term $\frac{\gamma}{\mu^{2}}\left(\rho_{v}+\rho_{\varepsilon}+m^{2}\rho_{x}\right)\to\infty$. Therefore, so $\lim_{\gamma\to 0}\frac{\partial(1/\lambda)}{\partial\mu}>0$.

(2) ρ_v is sufficiently small. As $\rho_v\to 0$, $\left[2m\left(\gamma/\mu\right)+m^{2}\rho_{x}-\left(\rho_{v}+\rho_{\varepsilon}\right)\right]$ $\to\left[2\frac{\rho_{\varepsilon}\left(\gamma^{2}+\mu\rho_{v}\rho_{x}+\mu\rho_{x}\rho_{\varepsilon}\right)}{\left(\gamma^{2}+\mu^{2}\rho_{v}\rho_{x}+\mu\rho_{x}\rho_{\varepsilon}\right)}+m^{2}\rho_{x}-\rho_{\varepsilon}\right]>\left[2\rho_{\varepsilon}+m^{2}\rho_{x}-\rho_{\varepsilon}\right]>0$, so we also have $\frac{\partial(1/\lambda)}{\partial\mu}>0$.

(3) ρ_ε is sufficiently small. As $\rho_\varepsilon\to 0$, $m=\frac{\mu\rho_{\varepsilon}\left(\gamma^{2}+\mu\rho_{v}\rho_{x}+\mu\rho_{\varepsilon}\rho_{x}\right)}{\gamma\left(\gamma^{2}+\mu^{2}\rho_{v}\rho_{x}+\mu\rho_{\varepsilon}\rho_{x}\right)}\to 0$ and $\frac{\partial m}{\partial\mu}=\frac{\rho_{\varepsilon}}{\gamma}\frac{\left(\gamma^{2}+\mu\rho_{\varepsilon}\rho_{x}\right)^{2}+\gamma^{2}\mu(2-\mu)\rho_{v}\rho_{x}+\mu^{2}\rho_{v}\rho_{\varepsilon}\rho_{x}^{2}}{\left(\gamma^{2}+\mu^{2}\rho_{v}\rho_{x}+\mu\rho_{\varepsilon}\rho_{x}\right)^{2}}\to 0$ and hence $\left[2m\left(\gamma/\mu\right)+m^{2}\rho_{x}-\left(\rho_{v}+\rho_{\varepsilon}\right)\right]\rho_{x}\frac{\partial m}{\partial\mu}\to 0$, but $\frac{\gamma}{\mu^{2}}\left(\rho_{v}+\rho_{\varepsilon}+m^{2}\rho_{x}\right)>0$.

C. Proof of Proposition 2

The first-order-condition for the exchange's decision problem is

$$\pi'\left(\mu\right)=\frac{1}{2\gamma}\left(\log\left[1+\frac{\gamma^{2}}{\mu^{2}\rho_{x}\left(\rho_{v}+\rho_{\varepsilon}\right)}\right]-\frac{2}{1+\frac{\gamma^{2}}{\mu^{2}\rho_{x}\left(\rho_{v}+\rho_{\varepsilon}\right)}}\frac{\gamma^{2}}{\mu^{2}\rho_{x}\left(\rho_{v}+\rho_{\varepsilon}\right)}\right).$$

Define

$$z\left(\mu\right)=\frac{\gamma^{2}}{\mu^{2}\rho_{x}\left(\rho_{v}+\rho_{\varepsilon}\right)}\in\left[\frac{\gamma^{2}}{\rho_{x}\left(\rho_{v}+\rho_{\varepsilon}\right)},\infty\right)$$

and its inverse function is

$$\mu\left(z\right)=\frac{\gamma}{\sqrt{z\rho_{x}\left(\rho_{v}+\rho_{\varepsilon}\right)}}.$$

So, we have

$$\pi'(\mu) = \frac{1}{2\gamma}\left[\log(1+z(\mu)) - \frac{2z(\mu)}{1+z(\mu)}\right].$$

Now let's first examine the properties of function $f(z) = \log(1+z) - \frac{2z}{1+z}$. We are going to show that (i) there is a unique positive solution $z^* \in (0,\infty)$ that solves the equation $f(z^*) = 0$, and that (ii) for $z \in (0, z^*)$, we have $f(z) < 0$, and for $z \in (z^*, \infty)$, we have $f(z) > 0$. Specifically, as z is close to zero, by first-order Taylor expansion, $f(z) \approx z - 2z = -z < 0$, and when z is large, $\lim_{z\to\infty} f(z) = \infty$. Thus, there will be a $z^* \in (0,\infty)$ such that $f(z^*) = 0$. We can easily solve for $z^* \approx 3.92$ with numerical method. Taking the first-order derivative of $f(z)$ delivers

$$f'(z) = \frac{z-1}{(1+z)^2}.$$

So for $z < 1$, $f(z)$ is decreasing and negative, since when z is small, $f(z)$ is negative. Then, as we increase z above 1, $f(z)$ will be increasing and it will cross 0 once, and then stay positive. Thus, we know that as long $0 < z < z^*$, then $f(z) < 0$, and when $z > z^*$, we have $f(z) > 0$.

So it is clear that if $\frac{\gamma^2}{\rho_x(\rho_v+\rho_\varepsilon)} > z^*$, then we must have $\pi'(\mu) > 0$ for all $\mu \in [0,1]$, because for the whole range of $z(\mu) \in [\frac{\gamma^2}{\rho_x(\rho_v+\rho_\varepsilon)}, \infty)$, z is greater than z^*, and as a result $\pi'(\mu) = f(z(\mu)) > 0$ and the maximum profit of the exchange is achieved at $\mu^* = 1$. If $\frac{\gamma^2}{\rho_x(\rho_v+\rho_\varepsilon)} < z^*$, then for $\mu \in (0, \mu(z^*))$, we know $z(\mu) \in (z^*, \infty)$ and hence $\pi'(\mu) = f(z(\mu)) > 0$; and for $\mu \in (\mu(z^*), 1)$, we have $z(\mu) \in (\frac{\gamma^2}{\rho_x(\rho_v+\rho_\varepsilon)}, z^*)$ and hence $\pi'(\mu) = f(z(\mu)) < 0$. So, the maximum is achieved at $\mu^* = \mu(z^*) = \frac{\gamma}{\sqrt{z^*\rho_x(\rho_v+\rho_\varepsilon)}}$. The optimal price q^* can be computed by substituting μ^* into the definition of $B(\mu)$ and using the condition of $f(z^*) = 0$.

D. Demand Function for Price Data with Endogenous Information

We first prove Proposition 6 and then characterize the three regions of the demand function for the exchange's price data. We also show that the positive variables—price informativeness, the cost of capital, return volatility, liquidity—become worse as the exchange increases the price q of price data.

Proof of Proposition 6 By equations (29)-(32),

$$\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}-\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}=\frac{\left(1-\beta\right)^2\frac{1}{\rho_v+\rho_\varepsilon}+\lambda^2\frac{1}{\rho_x}}{\frac{1}{\rho_v+\rho_\varepsilon+m^2\rho_x}}-\frac{\left(1-\beta\right)^2\frac{1}{\rho_v}+\lambda^2\frac{1}{\rho_x}}{\frac{1}{\rho_v+m^2\rho_x}}.$$

By Corollary 5, $m=\frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ when $\mu_s=0$. Substituting $m=\frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ and the expressions of β and λ (given by equations (25) and (26)) into the above equation, we have

$$\begin{aligned}&\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}-\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}\\ =&\frac{\gamma^2\rho_\varepsilon\left(\begin{array}{c}\gamma^4\rho_v+\gamma^4\rho_\varepsilon+\rho_x^2\rho_\varepsilon^3\mu_{ps}^4+\mu_p^2\rho_x^2\rho_\varepsilon^3\mu_{ps}^2+2\gamma^2\rho_x\rho_\varepsilon^2\mu_{ps}^2\\+2\mu_p\rho_x^2\rho_\varepsilon^3\mu_{ps}^3+2\gamma^2\mu_p\rho_x\rho_\varepsilon^2\mu_{ps}+2\gamma^2\rho_v\rho_x\rho_\varepsilon\mu_{ps}^2+2\gamma^2\mu_p\rho_v\rho_x\rho_\varepsilon\mu_{ps}\end{array}\right)}{\rho_x\left(\rho_v+\rho_\varepsilon\right)\left(\gamma^2\mu_p\rho_v+\rho_x\rho_\varepsilon^2\mu_{ps}^3+\gamma^2\rho_v\mu_{ps}+\gamma^2\rho_\varepsilon\mu_{ps}+\mu_p\rho_x\rho_\varepsilon^2\mu_{ps}^2\right)^2}>0.\end{aligned}$$

Thus, by equations (33)-(36), we know

$$\log\left(-E\left[V_s\left(\tilde{s}_i\right)\right]\right)-\log\left(-E\left[V_{ps}\left(\tilde{p},\tilde{s}_i\right)\right]\right)>\log\left(-E\left[V_u\right]\right)-\log\left(-E\left[V_p\left(\tilde{p}\right)\right]\right).$$

Of course, $\frac{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)}>\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p})}$ also implies $\frac{Var(\tilde{v}-\tilde{p}|\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)}>\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}$, and hence

$$\log\left(-E\left[V_p\left(\tilde{p}\right)\right]\right)-\log\left(-E\left[V_{ps}\left(\tilde{p},\tilde{s}_i\right)\right]\right)>\log\left(-E\left[V_u\right]\right)-\log\left(-E\left[V_s\left(\tilde{s}_i\right)\right]\right).$$

We next derive the demand function for price data in the three regions. In each region, we show that the equilibrium trader distributions are as described in the main text, and we also show that the positive variables get worse as the exchange raises q.

Region 1: The Outcomes for Low Values of q

We first determine the fraction of PS-traders in Region 1. When q is low, only PS- and P-informed traders are active (i.e., all traders will purchase price data), and by equations (29) and (30), the benefit for P-informed traders to acquire signal $\tilde{s}_i$ and become PS-informed traders is determined by:

$$\frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}\right]=\frac{1}{2\gamma}\log\left(1+\frac{\rho_\varepsilon}{\rho_v+m^2\rho_x}\right). \tag{44}$$

Since $\mu_s=0$, we have $m=\frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ by Corollary 5. Thus, equation (44) is the same as equation (37), the learning benefit function in Economy F. So, when $\underline{c}\leq c\leq\bar{c}$, the equilibrium fraction μ_{ps} of PS-informed traders is equal to μ_{ps}^F given by Proposition 5.

We can easily see that this result holds more generally as long as $\mu_s=0$, $\mu_{ps}>0$ and $\mu_p\geq 0$, because (i) $\mu_s=0$ implies $m=\frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ and (ii) "$\mu_{ps}>0$ and $\mu_p\geq 0$" implies that P-informed is indifferent between acquiring $\tilde{s}_i$ and not, and as a result, $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p}|\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)}\right]=c$ implies $\mu_{ps}=\mu_{ps}^F$. We summarize this result in the following proposition, which will be useful for our subsequent analysis.

Proposition 9 *Suppose $\underline{c}\leq c\leq\bar{c}$. When there are no S-informed traders and PS- and P-informed traders are active (i.e., $\mu_s=0$, $\mu_{ps}>0$ and $\mu_p\geq 0$), the equilibrium fraction of PS-informed traders in Economy D is the same as in Economy F; that is, $\mu_{ps}^*=\mu_{ps}^F=\frac{\gamma}{\rho_\varepsilon}\sqrt{\frac{\frac{\rho_\varepsilon}{e^{2\gamma c}-1}-\rho_v}{\rho_x}}$.*

The exchange can increase q to a threshold value of q_1 before it loses any customers. We next determine this value of q_1. Since P-informed traders have less incentive to keep price information than do PS-informed traders, we conjecture the threshold value q_1 is the extra benefit of P-informed traders keeping the price signal: $q_1 = \frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p})}\right]$ evaluated at $\mu_{ps} = \mu_{ps}^F$ and $\mu_p = 1 - \mu_{ps}^F$ (the equilibrium will not change since all traders still want to buy price data). By equations (31) and (32) and the facts of $\mu_{ps} + \mu_p = 1$ (which means that β and λ are only functions of two endogenous variables μ_{ps} and m by equations (25) and (26)) and of $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ (by Corollary 5), we have:

$$\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)} = 1 + \frac{\gamma^6\rho_v}{\rho_x\left(\gamma^2\rho_v + \gamma^2\mu_{ps}\rho_\varepsilon + \mu_{ps}^2\rho_\varepsilon^2\rho_x\right)^2}. \tag{45}$$

By Proposition 6, we know that μ_{ps} is fixed at the value of μ_{ps}^F when only PS- and P-informed traders are active, and thus, we have

$$q_1 = \frac{1}{2\gamma}\log\left[1 + \frac{\gamma^6\rho_v}{\rho_x\left[\gamma^2\left(\rho_v + \mu_{ps}^F\rho_\varepsilon\right) + \left(\mu_{ps}^F\right)^2\rho_\varepsilon^2\rho_x\right]^2}\right]. \tag{46}$$

Next, we check that PS- and P-informed traders will have no incentive to deviate along the process when the exchange raises the price q to q_1. That is, we want to ensure that the belief of the exchange is consistent. Given that both PS-informed and P-informed have the same ex ante utility, it suffices to examine any type of traders, say, P-informed traders and to show that $E\left[V_p\left(\tilde{p}\right)\right] > E\left[V_u\right]$ and $E\left[V_p\left(\tilde{p}\right)\right] > E\left[V_s\left(\tilde{s}_i\right)\right]$.

By Proposition 9, μ_{ps} is fixed at the value of μ_{ps}^F and thus the benefit of observing price information is fixed at the value of q_1. So, if $q < q_1$, then P-informed traders will be better-off if keeping the price information; that is,

$$E\left[V_p\left(\tilde{p}\right)\right] > E\left[V_u\right] \text{ or } \frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)} > e^{2\gamma q}.$$

By Proposition 6,

$$\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)} < \frac{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)} = e^{2\gamma c}.$$

Combining the above two inequalities, we have

$$\frac{\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p})}e^{-2\gamma q}}{\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}e^{-2\gamma c}} > \frac{e^{2\gamma q}e^{-2\gamma q}}{e^{2\gamma c}e^{-2\gamma c}} = 1 \Rightarrow E\left[V_p\left(\tilde{p}\right)\right] > E\left[V_s\left(\tilde{s}_i\right)\right],$$

by equations (35) and (34).

We summarize our results in the following lemma.

Lemma 1 *When $q \leq q_1$, all investors purchase the price information, and there are μ_{ps}^F fraction of PS-informed traders and $\left(1-\mu_{ps}^F\right)$ fraction of P-informed traders active in the market. The price informativeness is maintained at a constant level of $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$, and the equilibrium price coefficients are accordingly maintained at constants given by equations (25) and (26). The positive variables do not vary with q, given that the price functions do not change.*

Region 2: The Outcomes for Intermediate Values of q

Suppose the exchange would increases q above q_1. By Proposition 6, we know that the incentive of P-informed traders to keep price information is smaller than that of PS-informed traders, so that they will be the first to choose not to buy the price information. The exchange will charge a price q to make P-informed traders indifferent to uninformed. As q increases, more P-informed traders will become uninformed, and when q increases to a threshold value of q_2, all P-informed traders will switch to being uninformed traders. We next characterize the threshold value of q_2 and the distribution of trader types along the process of raising q toward q_2.

First, note that in the process of raising q toward q_2, Proposition 9 implies that the equilibrium fraction μ_{ps} of PS-informed traders is fixed at a constant of μ_{ps}^F, and hence by Corollary 5, the price informativeness is fixed at a constant of $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$, too.

Second, let's examine how q is determined. By equations (31), (32), (35) and (36), the benefit of observing price information for uninformed traders is

$$B_M\left(\mu_p\right) = \frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}\right] = \frac{1}{2\gamma}\log\left[\frac{\left(1-\beta\right)^2\rho_v^{-1}+\lambda^2\rho_x^{-1}}{\left(\rho_v+m^2\rho_x\right)^{-1}}\right], \tag{47}$$

which is a function of only one endogenous variable μ_p, because β and λ (which are given by equations (25) and (26)) are functions of three endogenous variables μ_p, μ_{ps} and m and the other two endogenous variables are fixed at constants, $\mu_{ps} = \mu_{ps}^F$ and $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$. In particular, since by equations (31), $Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right) = \frac{1}{\rho_v+m^2\rho_x}$, which is a constant $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$, how $B_M\left(\mu_p\right)$ changes with μ_p is completely determined by the behavior $Var\left(\tilde{v}-\tilde{p}\right) = \left(1-\beta\right)^2\rho_v^{-1}+\lambda^2\rho_x^{-1}$ (given by equation (32)), which can be shown to be decreasing with μ_p as follows.

By equation (26) and the facts of $\mu_{ps} = \mu_{ps}^F$ and of $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$, we have

$$\lambda = \frac{\gamma + \left(\mu_{ps}^F+\mu_p\right)\left(\frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}\right)\rho_x}{\mu_{ps}^F\rho_\varepsilon + \left(\mu_{ps}^F+\mu_p\right)\left(\rho_v + \left(\frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}\right)^2\rho_x\right)} \Rightarrow$$

$$\frac{\partial \lambda}{\partial \mu_p} = \frac{-\gamma \rho_v}{\left[\mu_{ps}^F \rho_\varepsilon + \left(\mu_{ps}^F + \mu_p\right)\left(\rho_v + \left(\frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}\right)^2 \rho_x\right)\right]^2} < 0. \tag{48}$$

Using the fact of $\beta = m\lambda = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}\lambda$ and equations (25) and (26), direct computation shows:

$$\begin{aligned}
\frac{\partial Var\left(\tilde{v} - \tilde{p}\right)}{\partial \mu_p} &= 2\left[\left(\beta - 1\right)\frac{1}{\rho_v}\frac{\mu_{ps}^F \rho_\varepsilon}{\gamma} + \lambda \frac{1}{\rho_x}\right]\frac{\partial \lambda}{\partial \mu_p} \\
&= \frac{2\gamma^3}{\rho_x\left[\gamma^2 \mu_p \rho_v + \gamma^2 \mu_{ps}^F \left(\rho_v + \rho_\varepsilon\right) + \left(\mu_p + \mu_{ps}^F\right)\left(\mu_{ps}^F\right)^2 \rho_\varepsilon^2 \rho_x\right]}\frac{\partial \lambda}{\partial \mu_p} < 0.
\end{aligned} \tag{49}$$

Therefore, the incentive of uninformed traders to acquire price information decreases with μ_p; that is, $B_M'\left(\mu_p\right) < 0$. As a result, the threshold value q_2 is set to be $B_M\left(0\right)$, which is given by equation (47) (and equations (25) and (26) and $\mu_{ps} = \mu_{ps}^F$ and $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$):

$$q_2 \equiv B_M\left(0\right) = \frac{1}{2\gamma}\log\left[1 + \frac{\gamma^6 \rho_v}{\left(\mu_{ps}^F\right)^2 \rho_x \left[\gamma^2\left(\rho_v + \rho_\varepsilon\right) + \left(\mu_{ps}^F\right)^2 \rho_\varepsilon^2 \rho_x\right]^2}\right]. \tag{50}$$

For any value of $q \in [q_1, q_2]$, there will be three types of traders active: PS-informed, P-informed, and uninformed. As the exchange increases q, the fraction of PS-informed is fixed at μ_{ps}^F, and the fraction of P-informed traders gradually decreases from $\left(1 - \mu_{ps}^F\right)$ toward 0, and the exact value of μ_p is determined by setting $B_M\left(\mu_p\right) = q$.

Third, let's show that in the process described above, all traders indeed have no incentives to deviate, so that we have an equilibrium (i.e., the belief of the exchange is consistent). Since all three active types of traders—PS-informed, P-informed, and uninformed—have the same ex ante utility, it is sufficient to show that any type of them has a higher utility than the remaining type, S-informed traders. Let us examine uninformed traders. Specifically, by

Proposition 6,

$$\log\left(-E\left[V_u\right]\right) - \log\left(-E\left[V_s\left(\tilde{s}_i\right)\right]\right) < \log\left(-E\left[V_p\left(\tilde{p}\right)\right]\right) - \log\left(-E\left[V_{ps}\left(\tilde{p}, \tilde{s}_i\right)\right]\right) = 0,$$

where the equality follows from the fact that PS-informed and P-informed have the same utility. Thus, $E\left[V_u\right] > E\left[V_s\left(\tilde{s}_i\right)\right]$.

Finally, we examine the implications of increasing q for positive variables. The price informativeness is maintained at a constant level of $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$. By equations (48) and (49), increasing q will decrease liquidity and increase return volatility through decreasing μ_p. The cost of capital is negatively related to the average precision across all investors, i.e.,

$$\frac{\mu_{ps}^F}{Var\left(\tilde{v} - \tilde{p} | \tilde{p}, \tilde{s}_i\right)} + \frac{\mu_p}{Var\left(\tilde{v} - \tilde{p} | \tilde{p}\right)} + \frac{1 - \mu_{ps}^F - \mu_p}{Var\left(\tilde{v} - \tilde{p}\right)}.$$

By equations (29) and (31) and the fact of $m = \frac{\mu_{ps}^F \rho_\varepsilon}{\gamma}$, we know that increasing q will not affect $\frac{1}{Var(\tilde{v} - \tilde{p} | \tilde{p}, \tilde{s}_i)}$ and $\frac{1}{Var(\tilde{v} - \tilde{p} | \tilde{p})}$. So,

$$\begin{aligned}
& \frac{\partial}{\partial q}\left[\frac{\mu_{ps}^F}{Var\left(\tilde{v} - \tilde{p} | \tilde{p}, \tilde{s}_i\right)} + \frac{\mu_p}{Var\left(\tilde{v} - \tilde{p} | \tilde{p}\right)} + \frac{1 - \mu_{ps}^F - \mu_p}{Var\left(\tilde{v} - \tilde{p}\right)}\right] \\
= \; & \frac{\partial}{\partial \mu_p}\left[\frac{\mu_p}{Var\left(\tilde{v} - \tilde{p} | \tilde{p}\right)} + \frac{1 - \mu_{ps}^F - \mu_p}{Var\left(\tilde{v} - \tilde{p}\right)}\right] \frac{\partial \mu_p}{\partial q} \\
= \; & \left[\left(\frac{1}{Var\left(\tilde{v} - \tilde{p} | \tilde{p}\right)} - \frac{1}{Var\left(\tilde{v} - \tilde{p}\right)}\right) + \left(1 - \mu_{ps}^F - \mu_p\right) \frac{\partial \frac{1}{Var(\tilde{v} - \tilde{p})}}{\partial \mu_p}\right] \frac{\partial \mu_p}{\partial q} < 0
\end{aligned}$$

because $\left(\frac{1}{Var(\tilde{v} - \tilde{p} | \tilde{p})} - \frac{1}{Var(\tilde{v} - \tilde{p})}\right) > 0$ (P-informed knows more than uninformed), $\frac{\partial \frac{1}{Var(\tilde{v} - \tilde{p})}}{\partial \mu_p} > 0$ (by equation (49)), and $\frac{\partial \mu_p}{\partial q} < 0$. Therefore, we have $\frac{\partial CC}{\partial q} > 0$; that is, increasing q will increase the cost of capital.

We summarize the results in Region 2 in the following lemma.

Lemma 2 *For any value of $q \in [q_1, q_2]$, there will be three types of traders active: PS-informed, P-informed, and uninformed. As the exchange increases q, the fraction of PS-informed is fixed at μ_{ps}^F, and the fraction of P-informed traders gradually decreases from $\left(1-\mu_{ps}^F\right)$ toward 0, and the exact value of μ_p is determined by setting $B_M\left(\mu_p\right) = q$. Along this process, price informativeness does not change, liquidity decreases, return volatility and the cost of capital increase.*

Region 3: The Outcomes for High Values of q: The Real Impact on Acquiring Fundamental Information $\tilde{s}_i$

When the price q reaches the value of q_2, all P-informed traders will switch to being uninformed. As the exchange continues to raise q, some of the PS-informed traders will not purchase the price data as well. As a result, only two types of traders, PS-informed and uninformed traders, are active in the market. Next, we characterize how the exchange sets price q, verify that no traders would like to deviate from their type so that the belief of the exchange is consistent, and characterize how increasing q affects the market outcomes.

Again, because $\mu_s = 0$, by Corollary 5, price informativeness is still given by $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$. By equations (25) and (26) in Proposition 4, the facts of $\mu_p = 0$ and $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$, we can express λ and β as a function of μ_{ps} only:

$$\lambda = \frac{\gamma + \mu_{ps}\frac{\mu_{ps}\rho_\varepsilon}{\gamma}\rho_x}{\mu_{ps}\left[\rho_v + \rho_\varepsilon + \left(\frac{\mu_{ps}\rho_\varepsilon}{\gamma}\right)^2\rho_x\right]}, \tag{51}$$

$$\beta = \frac{\frac{\mu_{ps}\rho_{\varepsilon}}{\gamma}\left(\gamma + \mu_{ps}\frac{\mu_{ps}\rho_{\varepsilon}}{\gamma}\rho_x\right)}{\mu_{ps}\left[\rho_v + \rho_{\varepsilon} + \left(\frac{\mu_{ps}\rho_{\varepsilon}}{\gamma}\right)^2\rho_x\right]}. \tag{52}$$

By equations equations (33) and (36), the joint benefit of observing both signals $\tilde{p}$ and $\tilde{s}_i$ is $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)}\right]$. Using equations(29) and (32) and the expressions of $m = \frac{\mu_{ps}\rho_{\varepsilon}}{\gamma}$ and λ and β (given by equations (51)-(52)), we can express the joint benefit of acquiring $\tilde{p}$ and $\tilde{s}_i$ as a function of μ_{ps} as follows:

$$B_H\left(\mu_{ps}\right) = \frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}\right] = \frac{1}{2\gamma}\log\left[1 + \frac{\gamma^2\left(\gamma^2 + \mu_{ps}^2\rho_{\varepsilon}\rho_x\right)}{\left[\gamma^2\left(\rho_v + \rho_{\varepsilon}\right) + \mu_{ps}^2\rho_{\varepsilon}^2\rho_x\right]\mu_{ps}^2\rho_x}\right]. \tag{53}$$

The exchange will charge the reservation value of PS-informed to set price q as follows:

$$q = B_H\left(\mu_{ps}\right) - c, \tag{54}$$

which is the difference between the joint benefit and the cost c of acquiring the fundamental signal $\tilde{s}_i$. We can show that the demand function is indeed decreasing, because

$$\frac{\partial}{\partial\mu_{ps}^2}\frac{\gamma^2\left(\gamma^2 + \mu_{ps}^2\rho_{\varepsilon}\rho_x\right)}{\left[\gamma^2\left(\rho_v + \rho_{\varepsilon}\right) + \mu_{ps}^2\rho_{\varepsilon}^2\rho_x\right]\mu_{ps}^2\rho_x} = -\frac{\gamma^2\left(\gamma^4\rho_v + \gamma^4\rho_{\varepsilon} + 2\gamma^2\mu_{ps}^2\rho_{\varepsilon}^2\rho_x + \mu_{ps}^4\rho_{\varepsilon}^3\rho_x^2\right)}{\rho_x\left[\left(\gamma^2\rho_v + \gamma^2\rho_{\varepsilon} + \mu_{ps}^2\rho_{\varepsilon}^2\rho_x\right)\mu_{ps}^2\right]^2} < 0.$$

We now verify that the belief of the exchange is consistent; that is, both PS-informed and uninformed traders have no incentive to deviate along the path specified above (the exchange increases the price q and μ_{ps} is set accordingly by equation (54)). It suffices to show that PS-informed are better-off than P-informed ($E\left[V_{ps}\left(\tilde{p},\tilde{s}_i\right)\right] > E\left[V_p\left(\tilde{p}\right)\right]$) and uninformed are better off than S-informed ($E\left[V_u\right] > E\left[V_s\left(\tilde{s}_i\right)\right]$).

First, by equations (29), (31), (33) and (35), the benefit of acquiring the fundamental signal $\tilde{s}_i$ for a potential P-informed trader to become PS-informed is:

$$\frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}\right]=\frac{1}{2\gamma}\log\left[1+\frac{\rho_\varepsilon}{\rho_v+m^2\rho_x}\right].$$

We also know that when $q=q_2$, we have $\mu_{ps}=\mu_{ps}^F$ and $m=\frac{\mu_{ps}^F\rho_\varepsilon}{\gamma}$, and at that moment, P-informed traders are just indifferent between staying P-informed versus becoming PS-informed, so that $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p}|\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)}\right]=c$ (which can also be verified by simply plugging the expression of μ_{ps}^F in Proposition 9). So, as the exchange increases q above q_2, so that μ_{ps} and m become smaller, the benefit of observing $\tilde{s}_i$ becomes larger for P-informed traders, and hence PS-informed traders are happy to paying both costs and seeing both signals.

Second, we examine whether uninformed traders want to become S-informed (we compare these two types of traders because the cost of accessing the signal is fixed at a constant of c). By equations (30), (32), (34) and (36) and the expressions of $m=\frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ and λ and β (given by equations (51)-(52)), the benefit of acquiring the fundamental signal $\tilde{s}_i$ for an uninformed trader is:

$$B_{SU}\left(\mu_{ps}\right)=\frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}|\tilde{s}_i\right)}\right]=\frac{1}{2\gamma}\log\left[1+\frac{\gamma^2\mu_{ps}^2\rho_v\rho_\varepsilon\rho_x}{\left(\gamma^2\left(\rho_v+\rho_\varepsilon\right)+\mu_{ps}^2\rho_\varepsilon^2\rho_x\right)\left(\gamma^2+\mu_{ps}^2\left(\rho_v+\rho_\varepsilon\right)\rho_x\right)}\right]. \tag{55}$$

Direct computation shows:

$$\frac{\partial B_{SU}\left(\mu_{ps}\right)}{\partial\mu_{ps}}\propto\left(\gamma^2-\mu_{ps}^2\rho_\varepsilon\rho_x\right). \tag{56}$$

So, when μ_{ps} is small, which tends to be the case when q is large, $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]$ is increasing in μ_{ps}. Alternatively, for sufficiently large γ, $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]$ is generally increasing in μ_{ps}.

We know that when $q = q_2$, we have $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right] < c$, since there is no S-informed traders active but uninformed traders are active. Then, as the exchange increases q and hence decreases μ_{ps}, $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]$ becomes even smaller (provided that $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]$ increases with μ_{ps}), and the uninformed traders will not want to acquire signal $\tilde{s}_i$ to become only S-informed.

Actually, we can further sharpen the lower bound of γ as follows. By condition (56), we know that $\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]$ achieves its maximum at $\mu_{ps} = \frac{\gamma}{\sqrt{\rho_x\rho_\varepsilon}}$ (by setting $\gamma^2 - \mu_{ps}^2\rho_\varepsilon\rho_x = 0$) and by equation (55), its maximum is

$$\max B_{SU}\left(\mu_{ps}\right) = B_{SU}\left(\frac{\gamma}{\sqrt{\rho_x\rho_\varepsilon}}\right) = \frac{1}{2\gamma}\log\left[\frac{(\rho_v+\rho_\varepsilon)(\rho_v+4\rho_\varepsilon)}{(\rho_v+2\rho_\varepsilon)^2}\right]. \tag{57}$$

We also know that the cost c falls in the range of $[\underline{c}, \bar{c}]$, where $\underline{c}$ and $\bar{c}$ are given by equations (39)-(38). Direct computation shows that $\bar{c} > \max B_{SU}\left(\mu_{ps}\right)$, and

$$\underline{c} \geq \max B_{SU}\left(\mu_{ps}\right) \Leftrightarrow \gamma \geq \frac{1}{2}\sqrt{\frac{\rho_v\rho_\varepsilon\rho_x}{\rho_v+\rho_\varepsilon}}.$$

Thus, as long as $\gamma \geq \frac{1}{2}\sqrt{\frac{\rho_v\rho_\varepsilon\rho_x}{\rho_v+\rho_\varepsilon}}$, we can ensure that $B_{SU}\left(\mu_{ps}\right) \leq \max B_{SU}\left(\mu_{ps}\right) \leq c$, so that uninformed traders have no incentive to acquire signal $\tilde{s}_i$ when the exchange raises q beyond q_2.

Finally we examine how the positive market outcomes change with an increase in q. Clearly, the price informativeness measure $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ decreases with q since μ_{ps} decreases with q. The return volatility increases with q, which can be shown as follows. By $\frac{\partial B_H\left(\mu_{ps}\right)}{\partial\mu_{ps}} = \frac{\partial}{\partial\mu_{ps}}\left(\frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{s}_i)}\right]\right) < 0$, we know $\frac{\partial}{\partial\mu_{ps}}\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)} < 0$. By equation (29) and $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$, we have $\frac{\partial}{\partial\mu_{ps}}\frac{1}{Var(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i)} > 0$. So it must be the case that $\frac{\partial Var(\tilde{v}-\tilde{p})}{\partial\mu_{ps}} < 0$. Since increasing q will decrease μ_{ps}, $Var\left(\tilde{v}-\tilde{p}\right)$ increases with q.

The cost of capital will increase with q as well. Recall that by Proposition 4, the cost of capital negatively relates to the average precision of traders' forecast:

$$\frac{\mu_{ps}}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}+\frac{1-\mu_{ps}}{Var\left(\tilde{v}-\tilde{p}\right)}.$$

Clearly,

$$\begin{aligned}
&\frac{\partial}{\partial\mu_{ps}}\left[\frac{\mu_{ps}}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}+\frac{1-\mu_{ps}}{Var\left(\tilde{v}-\tilde{p}\right)}\right]\\
=&\left[\frac{1}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}-\frac{1}{Var\left(\tilde{v}-\tilde{p}\right)}\right]\\
&+\mu_{ps}\frac{\partial}{\partial\mu_{ps}}\frac{1}{Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{s}_i\right)}+\left(1-\mu_{ps}\right)\frac{\partial}{\partial\mu_{ps}}\frac{1}{Var\left(\tilde{v}-\tilde{p}\right)}\\
>&\ 0.
\end{aligned}$$

So, $\frac{\partial CC}{\partial\mu_{ps}}<0$. Given that increasing q will decrease μ_{ps}, we know that CC increases with q.

As for liquidity, taking the direct derivative of equation (51) with respect to μ_{ps} shows:

$$\frac{\partial\lambda}{\partial\mu_{ps}}=-\frac{\gamma^4\left(\rho_v+\rho_\varepsilon+\rho_v\mu_{ps}\right)+\gamma^2\mu_{ps}^2\rho_\varepsilon\rho_x\left(-\rho_v+2\rho_\varepsilon+\mu_{ps}\rho_v\right)+\mu_{ps}^4\rho_\varepsilon^3\rho_x^2}{\gamma^3\mu_{ps}^2\left[\rho_\varepsilon+\rho_v+\left(\frac{\mu_{ps}\rho_\varepsilon}{\gamma}\right)^2\rho_x\right]^2}.$$

So, when γ is large or when ρ_v or ρ_ε is small, we have $\frac{\partial\lambda}{\partial\mu_{ps}}<0$, that is, increasing q will harm liquidity λ^{-1} through decreasing μ_{ps} ($\frac{\partial(1/\lambda)}{\partial q}=-\frac{1}{\lambda^2}\frac{\partial\lambda}{\partial\mu_{ps}}\frac{\partial\mu_{ps}}{\partial q}<0$).

We summarize the results in Region 3 in the following lemma.

Lemma 3 *When $q>q_2$ and when $\gamma>\frac{1}{2}\sqrt{\frac{\rho_v\rho_\varepsilon\rho_x}{\rho_v+\rho_\varepsilon}}$, there will be two types of traders, PS-informed and uninformed, active in the market. The fraction μ_{ps} of PS-informed traders is determined by equation (54): $q=B_H\left(\mu_{ps}\right)-c$. As the exchange increases q, price* *informativeness* $m = \frac{\mu_{ps}\rho_\varepsilon}{\gamma}$ *becomes smaller, return volatility* $\sigma(\tilde{v} - \tilde{p})$ *becomes larger, the cost of capital becomes larger, and if* γ *is large or* ρ_v *or* ρ_ε *is small, liquidity* λ^{-1} *becomes smaller as well.*

E. An Overlapping-Generations Model

In the main text, we use a one-period model as a shortcut to achieving a stationary analysis by assuming that the price that the exchange sells is the same as the price at which traders execute their orders. In this appendix, we examine the timing issue in more detail in a dynamic model where the exchange sells price data formed in previous periods and traders submit demand schedules and execute their orders at prevailing prices.

We find that our results in the main text hold in the dynamic model. First, there is a ranking in terms of positive variables in the three Economies (F better than D, and D better than E, where in economy E no one has price information).[21] That is, in Economy F, the price system is most informative and liquidity is the highest, while the cost of capital and return volatility are the lowest. Second, in terms of the fundamental information production, there is a "crowding-out" effect associated with the practice of selling price information. As a result, the total amount of fundamental information is the highest in Economy F and lowest in Economy E. This result is also driven by a complementarity effect of learning price information and fundamental information (although in a different form in the dynamic model). Third, the welfare of liquidity traders is the highest in Economy F and lowest in Economy E. So, moving from Economy F to Economy D, liquidity traders are losers and the exchange is the winner.

[21] In the main text we did not focus on economy E because if no one has contemporaneous price information there cannot be a market clearing price. We instead only examined it by taking the limit as the fraction of price informed traders converged to zero. Here as we are asking about past price information we can explicitly analyze economy E in which no one has past price information.

E1. The Setup

We consider an overlapping-generations (OLG) noisy rational expectations equilibrium model of the type that has become popular in the recent finance literature (e.g., Watanabe (2008), Biais et al. (2010), Andrei (2011), and Banerjee (2011)). To the extent that traders trade using demand schedules in each period in which they live, an OLG setup is necessary for past prices to be of any value, because in an infinite horizon model, traders can use dynamic demand schedules to back out the past prices and make decisions conditional on them. So, the OLG setup is a modeling device to capture the value of past prices and should not be interpreted literally. In effect, our setup can be equivalently interpreted as an economy where recurrent traders come on and off the market from time to time as long as they do not trade in two consecutive dates (they care about their wealth level when they leave the market).

Time is discrete and lasts forever. In each period, there is a continuum of rational traders who invest at date t and derive CARA expected utility of their wealth level at date $t+1$ when they exit the economy, where the absolute risk aversion coefficient is $\gamma > 0$. There are two tradable assets: a riskless bond and a stock. The riskless bond is assumed to be in infinitely elastic supply at a positive constant net interest rate $r_f > 0$. We need r_f to be positive to ensure that the stock price is bounded. The stock is a claim to a stream of dividends:

$$\tilde{d}_t = (1 - \phi_d)\, \bar{d} + \phi_d \tilde{d}_{t-1} + \tilde{v}_t, \tag{58}$$

where $\bar{d} > 0$ is the unconditional mean, $\phi_d \in (0, 1)$ is the persistence parameter, and

$$\tilde{v}_t \sim \text{i.i.d. } N\left(0, 1/\rho_v\right), \text{ with } \rho_v > 0, \tag{59}$$

is innovations. Liquidity traders supply $\tilde{x}_t$ units of the risky asset per capital in each period, which is unobservable and is specified as another AR(1) process:

$$\tilde{x}_t = (1 - \phi_x)\,\bar{x} + \phi_x \tilde{x}_{t-1} + \tilde{\eta}_t, \tag{60}$$

where $\bar{x} > 0$ is the unconditional mean, $\phi_x \in (0,1)$ is the persistence parameter, and

$$\tilde{\eta}_t \sim \text{i.i.d. } N\left(0, 1/\rho_\eta\right), \text{ with } \rho_\eta > 0, \tag{61}$$

is innovations.

Following the literature (e.g., Andrei (2011) and Banerjee (2011)), we assume that at each date, before trade occurs, each trader can spend a cost $c > 0$ to purchase a signal $\tilde{s}_t^i$ about the next period dividend innovation:

$$\tilde{s}_t^i = \tilde{v}_{t+1} + \tilde{\varepsilon}_t^i, \tag{62}$$

where

$$\tilde{\varepsilon}_t^i \sim \text{i.i.d. } N\left(0, 1/\rho_\varepsilon\right), \text{ with } \rho_\varepsilon > 0, \tag{63}$$

is independent of $\tilde{v}_{t+1}$ and where $\tilde{\varepsilon}_t^i$ is independent of $\tilde{\varepsilon}_t^j$ for $j \neq i$. Let $\tilde{p}_t$ be the ex dividend price at date t. Prices will aggregate information $\tilde{s}_t^i$. Traders submit demand schedules so they can condition on the information contained in price $\tilde{p}_t$.

The exchange determines the release of prices. For simplicity, we assume that the exchange must freely release all price information with a two period lag (because of regulation or because of information leakage), but it can charge a price of q for the last period price $\tilde{p}_{t-1}$. That is, at date t, all prices up to date $t-2$—$\{\tilde{p}_{t-2}, \tilde{p}_{t-3}, ...\}$—are public, and traders can pay a cost of $q > 0$ to see price $\tilde{p}_{t-1}$. For simplicity, we assume that dividends are known in real time, which seems reasonable since dividends are controlled by firms (who want to maximize transparency to minimize cost of capital). As a result, the public information at date t is dividends up to date t and prices up to date $t-2$:

$$\mathcal{I}_t^{public} = \{\tilde{d}_t, \tilde{d}_{t-1}, ...; \tilde{p}_{t-2}, \tilde{p}_{t-3}, ...\}. \tag{64}$$

Note that by the AR(1) specification of the dividend process (given by equation (58)), the innovations $\{\tilde{v}_t, \tilde{v}_{t-1}, ...\}$ are also known at date t. So, dividends $\tilde{d}_t$ and innovations $\tilde{v}_t$ are equivalent, and therefore, we can call both $\tilde{d}_t$ and $\tilde{v}_t$ "fundamentals".

As in the main text,, there are potentially four types of traders in the financial market at each date: PS-informed, S-informed, P-informed, and uninformed traders. PS-informed traders spend $(q+c)$ and observe both signals, so that their information set is: $\mathcal{I}_{t,i}^{ps} = \{\mathcal{I}_t^{public}, \tilde{p}_t, \tilde{p}_{t-1}, \tilde{s}_t^i\}$. This set includes the date-t price $\tilde{p}_t$ because traders submit demand schedules. Let $\mu_{ps} \geq 0$ denote the mass of PS-informed traders. Note that μ_{ps} does not depend on t because we will focus on a steady state of the economy. Similarly, S-informed traders have an information set $\mathcal{I}_{t,i}^s = \{\mathcal{I}_t^{public}, \tilde{p}_t, \tilde{s}_t^i\}$ and a mass of $\mu_s \geq 0$, P-informed traders have an information set of $\mathcal{I}_t^p = \{\mathcal{I}_t^{public}, \tilde{p}_t, \tilde{p}_{t-1}\}$ and a mass of $\mu_p \geq 0$, and uninformed traders have an information set of $\mathcal{I}_t^u = \{\mathcal{I}_t^{public}, \tilde{p}_t\}$ and a mass of $\mu_u \geq 0$. We have $\mu_{ps} + \mu_s + \mu_p + \mu_u = 1$. We will show that, as in the main text, all four types of traders cannot coexist in the market.

The exchange sets q to maximize its profit. The exchange's objective is the sum of its discounted profits, that is, $\Pi_t = E_t\left[\sum_s \frac{\pi_{t+s}}{\left(1+r_f\right)^s}\right]$,where π_t is its profit from selling price data on date t. The exchange will affect the trader type distribution through its control on q. In order to have a stationary economy, we will focus on economies where the exchange maintains a constant profit $\pi_t = \pi$ in each period (so that the endogenous trader distribution will be constant over time) and maximizes per period profit π.[22]

E2. Financial Market Equilibrium

At date t, the public information (dividends up to date t and past prices up to date $t-2$) will jointly reveal dividend innovations up to date t, $\{\tilde{v}_t, \tilde{v}_{t-1}, ...\}$, and supply innovations up to date $t-2$, $\{\tilde{\eta}_{t-2}, \tilde{\eta}_{t-1}, ...\}$. Given the AR(1) structure of the dividend and supply processes, it is natural to use $\tilde{d}_t$ and $\tilde{x}_{t-2}$ as state variables to avoid the infinite regress problem (see Bacchetta and Wincoop (2006)). So, we conjecture the price as a linear function of the state variables $(\tilde{d}_t, \tilde{x}_{t-2})$, aggregate dividend information $\tilde{v}_{t+1}$ (due to trading of fundamentally-informed traders), and supply innovations in the past two periods (due to liquidity trading $\tilde{x}_t$ on date t and traders' forecast on $\tilde{x}_t$):

$$\tilde{p}_t = \alpha_0 + \alpha_d \tilde{d}_t - \alpha_x \tilde{x}_{t-2} + \alpha_v \tilde{v}_{t+1} - \alpha_\eta \tilde{\eta}_t - \alpha_L \tilde{\eta}_{t-1}, \tag{65}$$

where all the coefficients of α's will be endogenously determined.

By the CARA-normal setup, each trader i' demand is:

$$D\left(\mathcal{I}_{t,i}\right) = \frac{E(\tilde{p}_{t+1} + \tilde{d}_{t+1}|\mathcal{I}_{t,i}) - \left(1 + r_f\right)\tilde{p}_t}{\gamma Var(\tilde{p}_{t+1} + \tilde{d}_{t+1}|\mathcal{I}_{t,i})}, \tag{66}$$

where $\mathcal{I}_{t,i}$ is the trader's information set, which includes the current price $\tilde{p}_t$, since traders submit demand schedules in a noisy rational expectations equilibrium. Thus, traders use

[22] Our stationary analysis restricts the action space of the exchange and therefore in principle, the exchange might deviate by charging different prices to different cohorts if by doing so it could generate a higher overall lifetime profit. But given the convexity structure of the economy, it seems natural for the exchange to maintain a constant profit over time, although we have not proven this conjecture.

their information to forecast the next period price plus dividend $\tilde{p}_{t+1}+\tilde{d}_{t+1}$, which by equations (58), (60) and (65) can be decomposed as follows:

$$\begin{aligned}\tilde{p}_{t+1}+\tilde{d}_{t+1} \quad = \quad & \underbrace{\alpha_0-\alpha_x\left(1-\phi_x\right)\bar{x}+\left(\alpha_d+1\right)\left(1-\phi_d\right)\bar{d}}_{\text{constant}} \\ & +\underbrace{\left(\alpha_d+1\right)\phi_d\tilde{d}_t-\alpha_x\phi_x\tilde{x}_{t-2}}_{\text{public information}} \\ & + \quad \underbrace{\left(\alpha_d+1\right)\tilde{v}_{t+1}-\alpha_x\tilde{\eta}_{t-1}-\alpha_L\tilde{\eta}_t}_{\text{forecastable from prices } \tilde{p}_t \text{ and } \tilde{p}_{t-1}\text{, and signal } \tilde{s}_t^i} \\ & + \underbrace{\left(\alpha_v\tilde{v}_{t+2}-\alpha_\eta\tilde{\eta}_{t+1}\right)}_{\text{unforecastable innovations}}. \end{aligned} \tag{67}$$

The interesting component in the above decomposition is the term $\left(\alpha_d+1\right)\tilde{v}_{t+1}-\alpha_x\tilde{\eta}_{t-1}-\alpha_L\tilde{\eta}_t$, which traders can use the signal $\tilde{s}_t^i$ and prices $\tilde{p}_t$ and $\tilde{p}_{t-1}$ to predict. This term is a combination of the "fundamental" $\tilde{v}_{t+1}$, which determines future dividends, and the supply innovations $\tilde{\eta}_{t-1}$ and $\tilde{\eta}_t$, which determine future risk premiums. Clearly, by equations (62) and (63), $\tilde{s}_t^i$ is useful for predicting $\tilde{v}_{t+1}$. By equation (65), the information contained in the current price $\tilde{p}_t$ and past price $\tilde{p}_{t-1}$ is useful for predicting $(\tilde{p}_{t+1}+\tilde{d}_{t+1})$ (or equivalently, $\left(\alpha_d+1\right)\tilde{v}_{t+1}-\alpha_x\tilde{\eta}_{t-1}-\alpha_L\tilde{\eta}_t$), and the information carried by these two prices respectively (joint with the public information) is:

$$\tilde{p}_t \quad \Leftrightarrow \quad \alpha_v\tilde{v}_{t+1}-\alpha_\eta\tilde{\eta}_t-\alpha_L\tilde{\eta}_{t-1}, \tag{68}$$

$$\tilde{p}_{t-1} \quad \Leftrightarrow \quad \tilde{\eta}_{t-1}. \tag{69}$$

Thus, unlike the signal $\tilde{s}_{t+1}^i$, the information content in past prices $\tilde{p}_{t-1}$ is about supply innovations $\tilde{\eta}_{t-1}$, not about the fundamental $\tilde{v}_{t+1}$. This highlights the conceptual difference between the role of past prices and that of the fundamental information in financial markets.

The market clearing condition at date t is that the total demand of rational traders for the stock is equal to the total supply provided by liquidity traders:

$$\int_0^{\mu_{ps}} D\left(\mathcal{I}_{t,i}^{ps}\right) di + \int_0^{\mu_s} D\left(\mathcal{I}_{t,i}^{s}\right) di + \mu_p D\left(\mathcal{I}_t^p\right) + \mu_u D\left(\mathcal{I}_t^u\right) = \tilde{x}_t. \tag{70}$$

To compute the equilibrium, we use equations (62), (63), (67), (68) and (69) and apply Bayes' rule to get expressions of $E(\tilde{p}_{t+1} + \tilde{d}_{t+1}|\mathcal{I}_{t,i})$ and $Var(\tilde{p}_{t+1} + \tilde{d}_{t+1}|\mathcal{I}_{t,i})$ in the demand function (equation (66)) for the four types of traders, and then use the market clearing condition (equation (70)) to solve for the equilibrium prices and compare coefficients with the conjectured price function (equation (65)) to get a fixed point, which is characterized by the following proposition.

Proposition 10 *The price coefficients in a stationary linear equilibrium price function*

$$\tilde{p}_t = \alpha_0 + \alpha_d \tilde{d}_t - \alpha_x \tilde{x}_{t-2} + \alpha_v \tilde{v}_{t+1} - \alpha_\eta \tilde{\eta}_t - \alpha_L \tilde{\eta}_{t-1}$$

are characterized by the following four equation system in terms of $(\alpha_x, \alpha_v, \alpha_\eta, \alpha_L)$*:*

$$\begin{cases} \alpha_x = \frac{\phi_x^2}{\varrho_A\left(1+r_f-\phi_x\right)}, \\ \alpha_v = \frac{\varrho_u\beta_u\alpha_v+\varrho_p\beta_p\alpha_v+\varrho_s\left[(\alpha_d+1)\frac{\rho_\varepsilon}{\rho_v+\rho_\varepsilon}+\beta_s\alpha_v\frac{\rho_v}{\rho_v+\rho_\varepsilon}\right]+\varrho_{ps}\left[(\alpha_d+1)\frac{\rho_\varepsilon}{\rho_v+\rho_\varepsilon}+\beta_{ps}\alpha_v\frac{\rho_v}{\rho_v+\rho_\varepsilon}\right]}{\varrho_A\left(1+r_f\right)}, \\ \alpha_\eta = \frac{\varrho_u\beta_u\alpha_\eta+\varrho_p\beta_p\alpha_\eta+\varrho_s\beta_s\alpha_\eta+\varrho_{ps}\beta_{ps}\alpha_\eta+1}{\varrho_A\left(1+r_f\right)}, \\ \alpha_L = \frac{\varrho_u\beta_u\alpha_L+\varrho_p\alpha_x+\varrho_s\beta_s\alpha_L+\varrho_{ps}\alpha_x+\phi_x}{\varrho_A\left(1+r_f\right)}, \end{cases}$$

where

$$
\begin{aligned}
\varrho_u &= \frac{\gamma^{-1}\mu_u}{\frac{\left(\alpha_L^2-\alpha_x\alpha_\eta\right)^2\rho_v\rho_\eta^{-1}+\left([\alpha_\eta(\alpha_d+1)-\alpha_v\alpha_L]^2+[\alpha_L(\alpha_d+1)-\alpha_x\alpha_v]^2\right)}{\left(\alpha_\eta^2+\alpha_L^2\right)\rho_v+\alpha_v^2\rho_\eta}+\left(\alpha_v^2\rho_v^{-1}+\alpha_\eta^2\rho_\eta^{-1}\right)}, \\
\varrho_p &= \frac{\gamma^{-1}\mu_p}{\frac{(\alpha_\eta(\alpha_d+1)-\alpha_v\alpha_L)^2}{\alpha_\eta^2\rho_v+\alpha_v^2\rho_\eta}+\left(\alpha_v^2\rho_v^{-1}+\alpha_\eta^2\rho_\eta^{-1}\right)}, \\
\varrho_s &= \frac{\gamma^{-1}\mu_s}{\frac{\left(\alpha_L^2-\alpha_x\alpha_\eta\right)^2(\rho_v+\rho_\varepsilon)\rho_\eta^{-1}+\left([\alpha_\eta(\alpha_d+1)-\alpha_v\alpha_L]^2+[\alpha_L(\alpha_d+1)-\alpha_x\alpha_v]^2\right)}{\left(\alpha_\eta^2+\alpha_L^2\right)(\rho_v+\rho_\varepsilon)+\alpha_v^2\rho_\eta}+\left(\alpha_v^2\rho_v^{-1}+\alpha_\eta^2\rho_\eta^{-1}\right)}, \\
\varrho_{ps} &= \frac{\gamma^{-1}\mu_{ps}}{\frac{(\alpha_\eta(\alpha_d+1)-\alpha_v\alpha_L)^2}{\alpha_\eta^2(\rho_v+\rho_\varepsilon)+\alpha_v^2\rho_\eta}+\left(\alpha_v^2\rho_v^{-1}+\alpha_\eta^2\rho_\eta^{-1}\right)}, \\
\varrho_A &= \varrho_u+\varrho_p+\varrho_s+\varrho_{ps},
\end{aligned}
$$

$$
\beta_u=\frac{\alpha_L\left(\alpha_x+\alpha_\eta\right)\rho_v+\alpha_v\left(\alpha_d+1\right)\rho_\eta}{\left(\alpha_\eta^2+\alpha_L^2\right)\rho_v+\alpha_v^2\rho_\eta},\beta_p=\frac{\alpha_\eta\alpha_L\rho_v+\alpha_v\left(\alpha_d+1\right)\rho_\eta}{\alpha_\eta^2\rho_v+\alpha_v^2\rho_\eta},
$$

$$
\beta_s=\frac{\alpha_L\left(\alpha_x+\alpha_\eta\right)\left(\rho_v+\rho_\varepsilon\right)+\alpha_v\left(\alpha_d+1\right)\rho_\eta}{\left(\alpha_\eta^2+\alpha_L^2\right)\left(\rho_v+\rho_\varepsilon\right)+\alpha_v^2\rho_\eta},\beta_{ps}=\frac{\alpha_\eta\alpha_L\left(\rho_v+\rho_\varepsilon\right)+\alpha_v\left(\alpha_d+1\right)\rho_\eta}{\alpha_\eta^2\left(\rho_v+\rho_\varepsilon\right)+\alpha_v^2\rho_\eta}.
$$

The other two coefficients are:

$$
\alpha_d=\frac{\phi_d}{1+r_f-\phi_d},\alpha_0=\frac{1+r_f}{1+r_f-\phi_d}\frac{\left(1-\phi_d\right)\bar{d}}{r_f}-\frac{1+r_f\left(1+\phi_x\right)}{\phi_x^2}\frac{\left(1-\phi_x\right)\alpha_x\bar{x}}{r_f}.
$$

E3. The Results

Parameter Values and Computation Methodology

As in the main text, we are interested in comparing three economies: (i) Economy D, where the profit-maximizing exchange sells the previous period price $\tilde{p}_{t-1}$ to traders at an endogenous price q^*; (ii) Economy F, where $\tilde{p}_{t-1}$ is freely disclosed to all traders; and (iii)

Economy E, where no one knows $\tilde{p}_{t-1}$. As standard in the literature, the complexity of the fixed point problem in the OLG economy precludes simple analytical analysis. We therefore use numerical analysis to examine the implications of selling price information.

There are ten exogenous parameters in our economy: $\left(r_f, \bar{d}, \phi_d, \rho_v, \rho_\varepsilon, c, \bar{x}, \phi_x, \rho_\eta, \gamma\right)$. The parameters chosen are given in Panel A of Table A1. For the technology parameters except the AR(1) coefficient ϕ_x of the liquidity trading, we borrow the values of Andrei (2011) and Banerjee (2011) that are picked to match the monthly returns of the market portfolio over the period January 1983 to December 2008. Unlike Andrei (2011) and Banerjee (2011) who set $\phi_x = 0$ so that the liquidity trading process $\tilde{x}_t$ is i.i.d. over time, we instead set $\phi_x = 0.5$, because as equation (69) shows, the value of knowing $\tilde{p}_{t-1}$ is to forecast liquidity trading shocks, so if $\tilde{x}_t$ was i.i.d., there would be no value for the past price $\tilde{p}_{t-1}$, which is not suitable for our study. For the additional technology parameter c, we set it at 0.218, which was chosen so that the fraction of informed traders is interior in both Economies E and F.[23] We set the risk aversion parameter γ at 0.1. The literature on time-dependent risk aversion suggests that risk aversion increases with horizon length (e.g., Bommier and Rochet, 2006). So, to the extent that our economy is interpreted as in a high-frequency trading context, a low risk aversion parameter of 0.1 seems reasonable in a calibrated economy. In addition, this small value of γ guarantees that the convergence of our algorithm is very fast.

[INSERT TABLE A1 HERE]

Given any cost c of acquiring signal $\tilde{s}_t^i$, for Economies E and F, we need to determine the equilibrium fractions (μ_s^E and μ_{ps}^F respectively) of traders acquiring the signal $\tilde{s}_t^i$ (and then we can use Proposition 10 to determine the corresponding financial market equilibrium). We find these fractions using a grid-search method. For example, for Economy E, we form a grid

[23] Our results are robust to alternative values of $\phi_x > 0$ and $c > 0$.

of μ_s on $[0,1]$, then for any given μ_s, we first use Proposition 10 to compute the coefficients of the price function, then use these coefficients to compute the benefit of observing the signal $\tilde{s}_t^i$, that is, $\frac{1}{2\gamma\left(1+r_f\right)} \log \left[\frac{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_t^u\right)}{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_{t,i}^s\right)}\right]$, and finally find the value of $\mu_s = \mu_s^E$ that equates the benefit and the cost c. Note that the system of coefficients in Proposition 10 maps the conjectured coefficients to the implied coefficients. So, we employ this property and use iterations to find the equilibrium coefficients, that is, for any starting values for the coefficients, we compute the right hand side of the system to define the new values, and continue this process until it converges. This method guarantees that the computed financial equilibrium is stable.

For Economy D, the basic computation idea is the following. First, we identify how trader distributions vary with the price q of past prices, and accordingly determine the demand for the price data and the profit-maxmizing price q^*. Second, we employ the properties of trader distributions driving the demand function for the past prices to compute the equilibrium trader type distribution corresponding to the optimal q^*. Finally, we use Proposition 10 to determine the financial market equilibrium corresponding to the equilibrium trader type distribution. We now turn to the analysis of the demand for the past price data.

Demand for the Past Price Data and the Crowding-Out Effect

Our numerical analysis suggests that as the exchange gradually increases price q of past price data from 0 to an endogenous threshold value q_3, the economy gradually moves from Economy F where all traders observe $\tilde{p}_{t-1}$ to Economy E where no one purchases and observes $\tilde{p}_{t-1}$. Specifically, trader distributions change with q as follows.

There exist three endogenous threshold values q_1, q_2, and q_3 (with $0 < q_1 < q_2 < q_3$) such that:

(1) For $q \leq q_1$, PS- and P-informed traders are active and their masses are maintained as those values in Economy F, that is, $\mu_{ps} = \mu_{ps}^F$, $\mu_p = \mu_p^F$ and $\mu_{ps} + \mu_p = 1$;

(2) For $q_1 < q < q_2$, PS-, P- and S-informed traders are active, and when q gradually increases from q_1 to q_2, some PS-informed become P-informed and some PS-informed traders become S-informed traders, and when $q = q_3$, no PS-informed traders are active; that is, there exist two constants $\mu_{p2} > \mu_p^F$ and $\mu_{s2} > 0$, such that "q: $q_1 \nearrow q_2 \Rightarrow \mu_{ps}$: $\mu_{ps}^F \searrow 0$, μ_p: $\mu_p^F \nearrow \mu_{p2}$, and μ_s: $0 \nearrow \mu_{s2}$;"

(3) For $q_2 \leq q < q_3$, P- and S-informed and uninformed traders are active, and when q gradually increases from q_2 to q_3, some of P- and S-informed traders switch to uninformed traders, and when $q = q_3$, no P-informed traders are active and the masses of S-informed and uninformed traders are the same as those values in Economy E; that is, "q: $q_2 \nearrow q_3 \Rightarrow \mu_p$: $\mu_{p2} \searrow 0$, μ_s: $\mu_{s2} \searrow \mu_s^E$, μ_u: $0 \nearrow \mu_u^E$;"

(4) For $q \geq q_3$, S-informed and uninformed traders are active and their masses are maintained as those values in Economy E, that is, $\mu_s = \mu_s^E$, $\mu_u = \mu_u^E$ and $\mu_s + \mu_u = 1$.

The intuition for Regions (1) and (4) is straightforward: When price q is sufficiently low or high, all or no traders will purchase the past price data, so that the trader distribution is identical to those in Economy F and Economy E, respectively.

The intuition for Regions (2) and (3) is driven by two facts. First, in our OLG model, the value of $\tilde{p}_{t-1}$ is lower for S-informed traders (who observe both $\tilde{p}_t$ and $\tilde{s}_t^i$) than for uninformed traders (who only observe $\tilde{p}_t$), and as a result, when q goes above q_1 in Region (2), it is PS-informed traders who first stop purchasing price data. This explains why PS-informed traders switch types in Region (2). This result differs from our one-period model in the main text where the price data sold by the exchange is the same as the execution price of the submitted orders. In the present dynamic model, price data is the past price and all traders can observe prevailing prices (as they submit demand schedules), which contain information about future prices and dividends. As suggested by Proposition 7.1 of Admati and Pfleiderer (1986), the value of an additional signal ($\tilde{p}_{t-1}$) to a trader who observes only the equilibrium price ($\tilde{p}_t$) is typically higher than its value to a trader who observes both price ($\tilde{p}_t$) and another signal ($\tilde{s}_t^i$).

Second, similar to our one-period model in the main text, there is still a complementarity effect in observing $\tilde{p}_{t-1}$ and $\tilde{s}_t^i$, albeit in a different form and through a different channel: An increase in the mass of traders acquiring one signal will lead more traders to acquire the other signal. This explains why S- and P-informed traders simultaneously increase in Region (2) and simultaneously decrease in Region (3). This complementarity effect is driven by the endogenous price informativeness of the current prices and has been systematically studied by Goldstein and Yang (2011): By equations (62) and (69), the signal $\tilde{s}_t^i$ owned by S-informed traders is about $\tilde{v}_{t+1}$ and the signal $\tilde{p}_{t-1}$ owned by P-informed traders is about $\tilde{\eta}_{t-1}$, and both $\tilde{v}_{t+1}$ and $\tilde{\eta}_{t-1}$ affect traders' future payoffs (equation (67)). So, when there are more S-informed traders in the market, the current price $\tilde{p}_t$ will incorporate more of their information and reveal $\tilde{v}_{t+1}$ to a greater extent, and therefore reduce the uncertainty faced by P-informed traders, which induces more uninformed traders to become P-informed as well.

The pattern of trader type distributions varying with the price q has two important implications. First, it directly determines the demand for the exchange's past price data. The demand function is downward sloping, and the exchange will choose a value of $q^* \in [q_1, q_2]$ to maximize its profit.

Second, similar to our one-period model in the main text, the practice of selling past prices has a "crowding-out effect", that is, the total amount of fundamental information $\mu_{ps} + \mu_s$

in Economy D is lower than that in Economy F and higher than that in Economy E. This result is driven by the aforementioned complementarity effect in observing $\tilde{p}_{t-1}$ and $\tilde{s}_t^i$, which is reflected in Regions (2) and (3). To see this point more clearly, note that as the exchange increases q from 0 to q_3, the total amount of fundamental information continuously decreases from μ_{ps}^F to μ_s^E. Specifically, when $q = 0$, the economy corresponds to Economy F, and the total amount of fundamental information is μ_{ps}^F. As the exchange increases q into Region (2), some PS-informed switch to S-informed, and some PS-informed switch to P-informed, and this new added P-informed traders reduce the total mass of observing signals $\tilde{s}_t^i$. As the exchange further increases q into Region (3), the total amount fundamental information is μ_s and is decreasing in q, because P-informed decreases with q and there is a complementarity effect between becoming S-informed and P-informed. When q reaches the value of q_3, the total amount of fundamental information reaches its minimum μ_s^E.

Next, we briefly discuss how the threshold values of q_1, q_2 and q_3 are determined and how they can be used in our numerical computation. The value of q_1 is determined as the marginal benefit of seeing price data for PS-informed traders when $\mu_{ps} = \mu_{ps}^F$ and $\mu_p = \mu_p^F$, that is, $q_1 = \frac{1}{2\gamma\left(1+r_f\right)} \log\left[\frac{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_t^p\right)}{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_{t,i}^{ps}\right)}\right]$. To determine q_2, note that when $q = q_2$, only P- and S-informed are active ($\mu_{p2} + \mu_{s2} = 1$), and PS-informed traders are just indifferent to S-informed traders. Therefore, we can use the equation of $\frac{1}{2\gamma\left(1+r_f\right)} \log\left[\frac{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_t^p\right)}{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_{t,i}^{ps}\right)}\right] = c$, which is a function of μ_{p2} only, to determine the threshold value of μ_{p2}, and then set $q_2 = \frac{1}{2\gamma\left(1+r_f\right)} \log\left[\frac{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_{t,i}^s\right)}{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_{t,i}^{ps}\right)}\right]$. When $q = q_3$, there are only S-informed and uninformed traders ($\mu_s = \mu_s^E$), and we know that P-informed are indifferent to uninformed traders. So, we can set $q_3 = \frac{1}{2\gamma\left(1+r_f\right)} \log\left[\frac{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_t^u\right)}{Var\left(p_{t+1}+\tilde{d}_{t+1}|\mathcal{I}_t^p\right)}\right]$, where the variances are computed using the equilibrium coefficients in the price function when $\mu_s = \mu_s^E$ and $\mu_u = \mu_u^E$.

Once we know the values of q_1, q_2 and q_3, we can explicitly form grids on q in Regions

(2) and (3) (which are the relevant regions for the exchange's profit-maximization problem), and for each q, we use the property that all active traders have the same indirect utility to solve for the corresponding trader distributions, which in turn determine the demand for the exchange's price data and hence the optimal q^* that maximizes the exchange's profit.

The Impact of Selling Past Prices

Panel B of Table A1 presents the implications of selling past price data. The magnitudes are comparable to those reported in Andrei (2011). The result is representative in the sense that as we change exogenous parameter values, the rankings in the three economies are very similar, although the magnitudes are different.

We can see that all of our results in the one-period model in the main text hold in the present OLG economy: (i) In terms of positive variables, the ranking is that Economy F is better than Economy D better than Economy E, that is, Economy F has the highest price informativeness and liquidity, and it has the lowest cost of capital and return volatility. (ii) Due to the "crowding-out effect" of selling price data, there is a ranking of total amount of fundamental information $\tilde{s}_t^i$: Economy F has a greater amount of information $\tilde{s}_t^i$ than Economy D, which in turn has a greater amount than Economy E. (iii) In terms of the welfare of liquidity traders and the total welfare, we have the same ranking, that is, liquidity traders are best-off in Economy F. By construction, the exchange is still best-off in Economy D. Rational traders are best-off in Economy E. Interestingly, they are better-off in Economy F than in Economy D, because their saving on the purchase of past price data exceeds their loss of trading in the relatively transparent economy.

Finally, we discuss how these variables are computed in this dynamic economy. Clearly, the total amounts of fundamental information in the three economies are μ_s^E, $\mu_{ps}^D + \mu_s^D$ and

μ_{ps}^F, respectively. For the price informativeness, we still use the ratio of

$$m = (\alpha_v / \alpha_\eta), \tag{71}$$

because the price and dividend history till date t is equivalent to a signal of $\alpha_v \tilde{v}_{t+1} - \alpha_\eta \tilde{\eta}_t$ and therefore the ratio of (α_v/α_η) reflects the informativeness about the fundamental $\tilde{v}_{t+1}$. We still use the inverse of price impact of liquidity traders, that is, $\left(\frac{\partial \tilde{p}_t}{\partial \tilde{x}_t}\right)^{-1}$, to capture liquidity. By the price function (equation (65)) and the AR(1) process of $\tilde{x}_t$ (equation (60)), we have

$$liquidity = \frac{1}{\partial \tilde{p}_t / \partial \tilde{x}_t} = \frac{1}{\partial \tilde{p}_t / \partial \tilde{\eta}_t} = \frac{1}{\alpha_\eta}. \tag{72}$$

We follow Andrei (2011) and Banerjee (2011) in defining returns on the risky asset as

$$\tilde{R}_{t+1} = \tilde{p}_{t+1} + \tilde{d}_{t+1} - (1 + r_f)\, \tilde{p}_t, \tag{73}$$

and use equations (58) and (65) to compute its volatility as the return volatility:

$$\sigma\left(\tilde{R}_{t+1}\right) = \sqrt{\begin{array}{c} \left(\frac{\left[(\alpha_d+1)\phi_d - \left(1+r_f\right)\alpha_d\right]^2}{1-\phi_d^2} + \left[(\alpha_d + 1) - (1 + r_f)\,\alpha_v\right]^2 + \alpha_v^2\right)\rho_v^{-1} \\ + \left(\frac{\alpha_x^2\left[\phi_x - \left(1+r_f\right)\right]^2}{1-\phi_x^2} + \left[\alpha_x - (1 + r_f)\,\alpha_L\right]^2 + \left[\alpha_L - (1 + r_f)\,\alpha_\eta\right]^2 + \alpha_\eta^2\right)\rho_\eta^{-1} \end{array}}. \tag{74}$$

Regarding the cost of capital, we have two alternatives, which turn out to be identical up to a scalar. First, we can use the expected return, $E(\tilde{R}_{t+1})$. By Proposition 10, we can compute

$$E(\tilde{R}_{t+1}) = \frac{1 + r_f - \phi_x}{\phi_x^2} \alpha_x \bar{x}. \tag{75}$$

Second, we can define cost of capital as the difference between some fundamental value of the risky asset and the current prevailing price $\tilde{p}_t$. Given the exogenous risk-free rate, it is natural to define the fundamental value of the risky asset as the sum of the discounted future dividends, that is:

$$\tilde{F}_t = E_t \sum_{s=1}^{\infty} \left[\frac{\tilde{d}_{t+s}}{(1+r_f)^s} \right] = \frac{\bar{d}}{r_f} + \frac{\phi_d(\tilde{d}_t - \bar{d})}{1 + r_f - \phi_d}. \tag{76}$$

So, we can alternatively define the cost of capital as $E(\tilde{F}_t - \tilde{p}_t)$. By Proposition 10, we can compute it and find that it is proportional to $E(\tilde{R}_{t+1})$:

$$E(\tilde{F}_t - \tilde{p}_t) = \frac{1 + r_f - \phi_x}{r_f \phi_x^2} \alpha_x \bar{x} = \frac{E(\tilde{R}_{t+1})}{r_f}. \tag{77}$$

Thus, the two definitions are equivalent. Table A1 reports $E(\tilde{R}_{t+1})$ as the cost of capital.

We still use the certainty equivalent of the indirect utility of rational traders to represent their welfare. That is,

$$\begin{aligned} WEL_R &= -\frac{1}{\gamma} \log \left(-E \left[V \left(\mathcal{I}_{t,i} \right) \right] \right) \\ &= \frac{1}{2\gamma} \log \left[\frac{Var(\tilde{R}_{t+1})}{Var(\tilde{R}_{t+1} | \mathcal{I}_{t,i})} \right] + \frac{\left[E(\tilde{R}_{t+1}) \right]^2}{2\gamma Var(\tilde{R}_{t+1})} - (1 + r_f)\, q, \end{aligned} \tag{78}$$

where $\mathcal{I}_{t,i}$ is the information set of a typical active rational trader.

The welfare of liquidity traders is defined as $-E[(\tilde{F}_t - \tilde{p}_t)\tilde{x}_t]$, which is still the negative of the expected opportunity cost of satisfying some unmodelled exogenous liquidity need associated with a trade of $\tilde{x}_t$ shares relative to a buy-and-hold strategy. By equations (65), (60), (76) and (77), we have:

$$
\begin{aligned}
WEL_L &= -E[(\tilde{F}_t - \tilde{p}_t)\tilde{x}_t] \\
&= -\bar{x}E(\tilde{F}_t - \tilde{p}_t) - Cov(\tilde{F}_t - \tilde{p}_t, \tilde{x}_t) \\
&= -\bar{x}\frac{E(\tilde{R}_{t+1})}{r_f} - \left(\frac{\alpha_x \phi_x^2}{1-\phi_x^2} + \alpha_\eta + \alpha_L \phi_x\right)\rho_\eta^{-1}, \qquad (79)
\end{aligned}
$$

which decreases with the cost of capital $E(\tilde{R}_{t+1})$ and illiquidity α_η, consistent with our one-period model in the main text.

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